Discrete Tomography in the real world

Tomography & Applications: Discrete Tomography and Image Reconstruction

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Overview

Discrete Tomography

- Causes of insufficient data
- DART
- Some examples

Reality check

- Prior knowledge
- Material imperfections
- Partial volume effect
- Projection noise
- Beam hardening

Improving DART

- Grey level estimation
- softDART
- Spatial Coherence Prior

Notation

$$\boldsymbol{W}\boldsymbol{v} = \boldsymbol{p}$$

- $\boldsymbol{v} \in \mathbb{R}^n$: reconstruction volume
- $\boldsymbol{v}^* \in \mathbb{R}^n$: real volume
- $\boldsymbol{p} \in \mathbb{R}^m$: projection data
- $\mathbf{I} \in \mathbb{N}^m$: photon counts (after flat field correction)
- $\boldsymbol{W} \in \mathbb{R}^{m \times n}$: projection matrix
- m: number of measurements
- n: number of voxels

$$\boldsymbol{p} = -\log\left(\frac{\boldsymbol{I}}{I_0}\right)$$

Number of measurements

Sufficient projection data

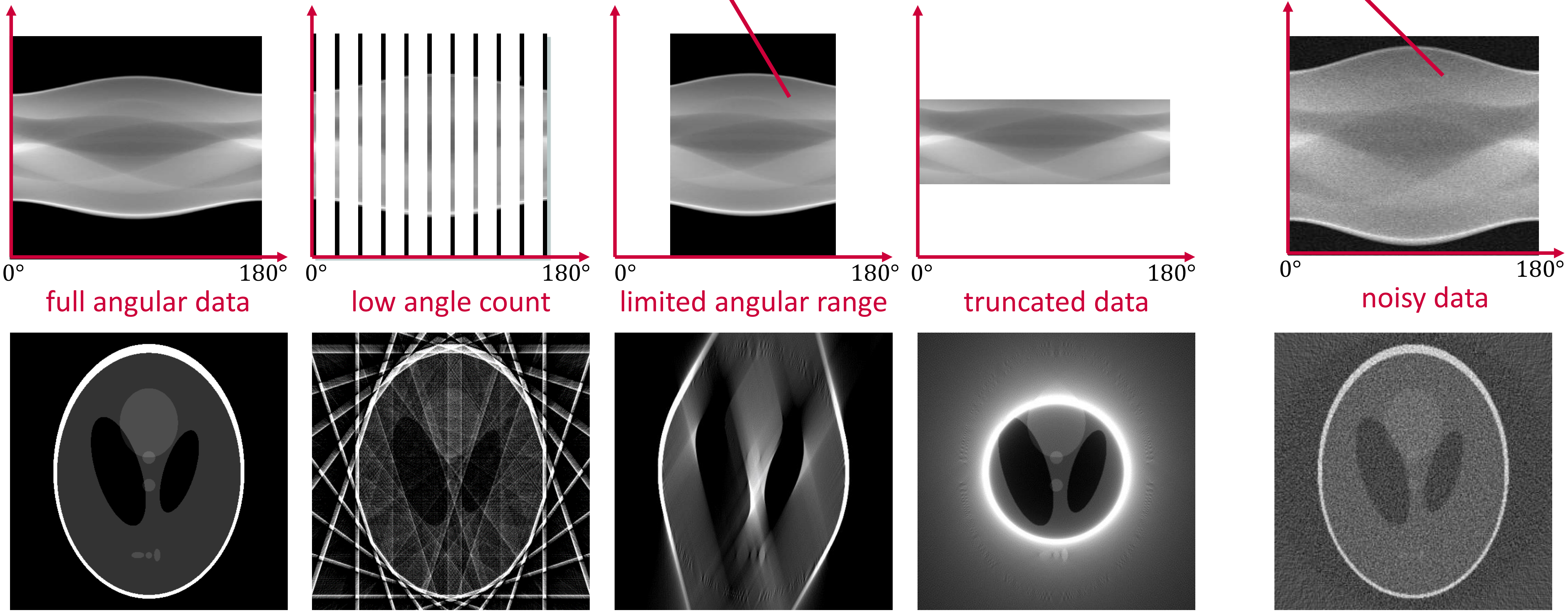
Insufficient projection data

Insufficient projection data

$$D_i = I_0 \exp\left(-\sum_j w_{ij} v_j^*\right)$$

$$\Pr(I_i|v^*) = \Pr(I_i = D_i) = 1$$

$$\Pr(I_i|v^*) = \frac{D_i}{I_i!}\exp(-D_i)$$

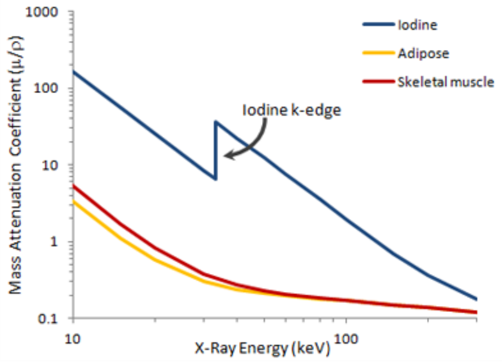


Number of measurements

Sufficient projection data

Insufficient projection data

Insufficient projection data
+ prior knowledge

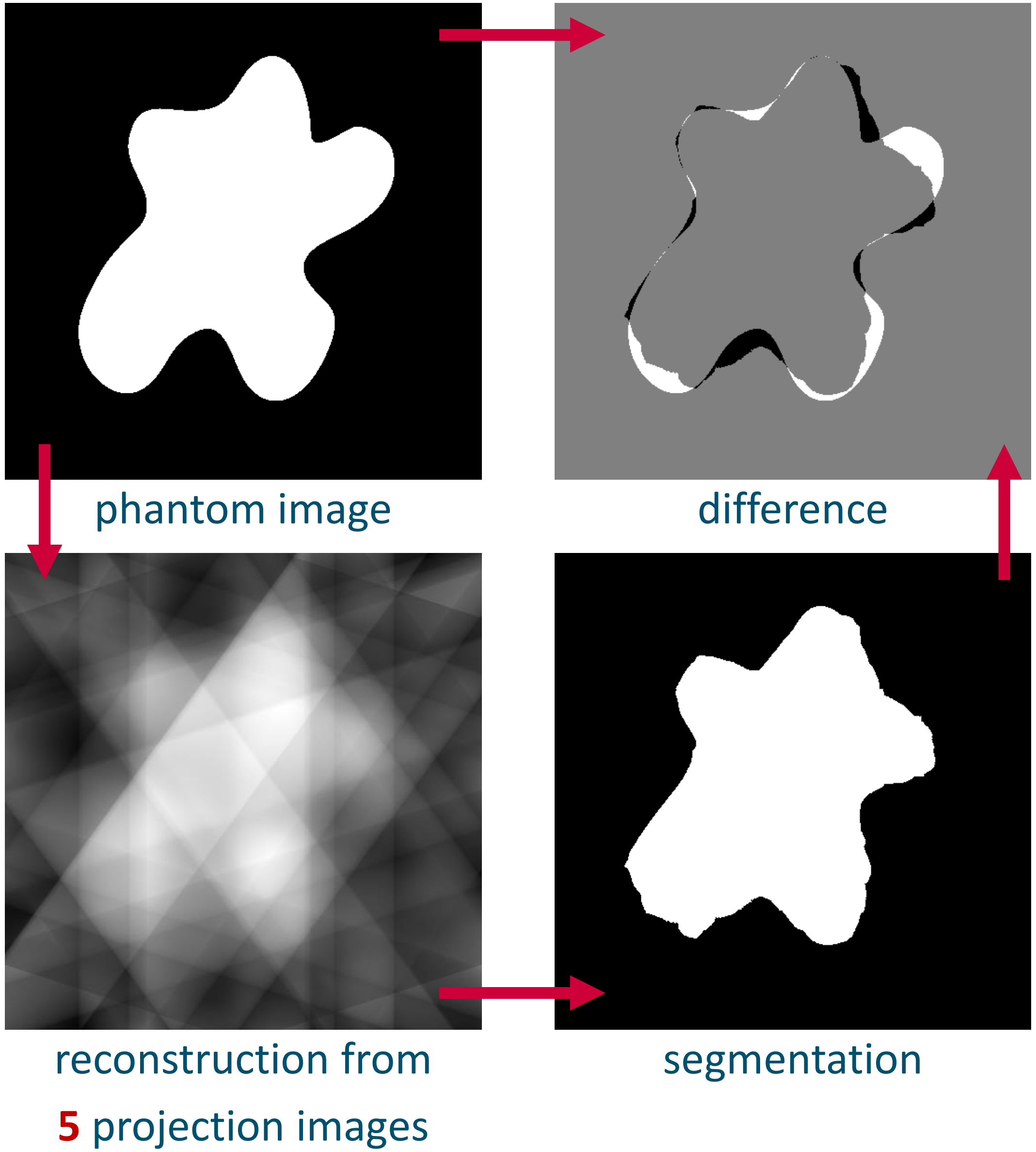


attenuation coefficients

sparsity

CAD model

An observation

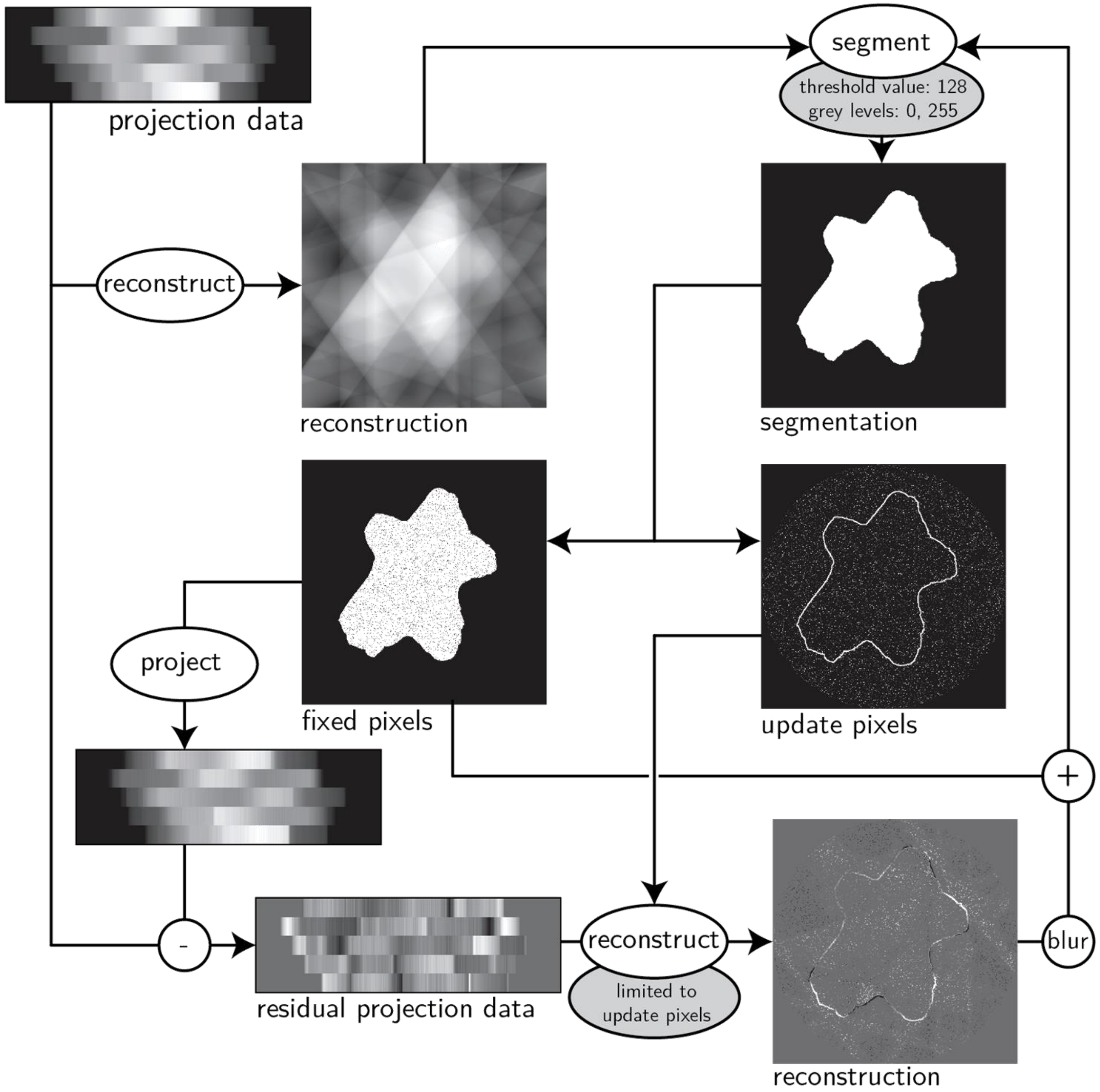


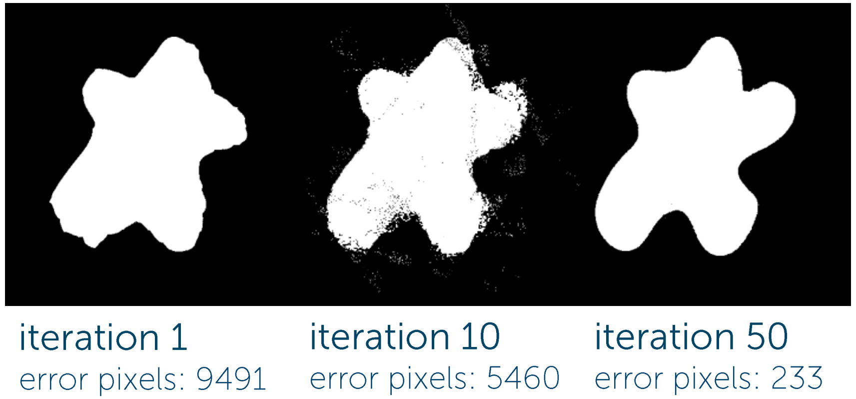
Discrete
Algebraic
Reconstruction
Technique
=> DART

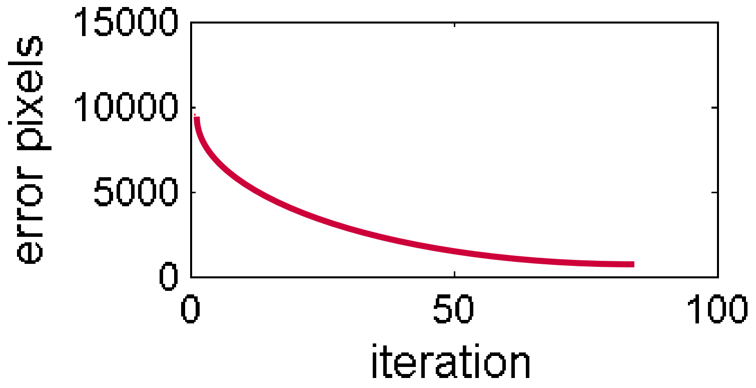
Assumptions:

- homogeneous objects
- prior knowledge: grey levels

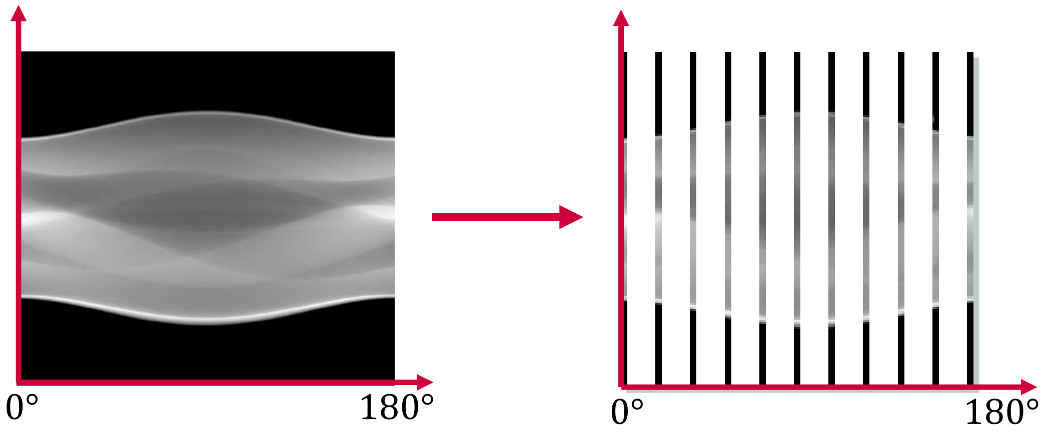
DART

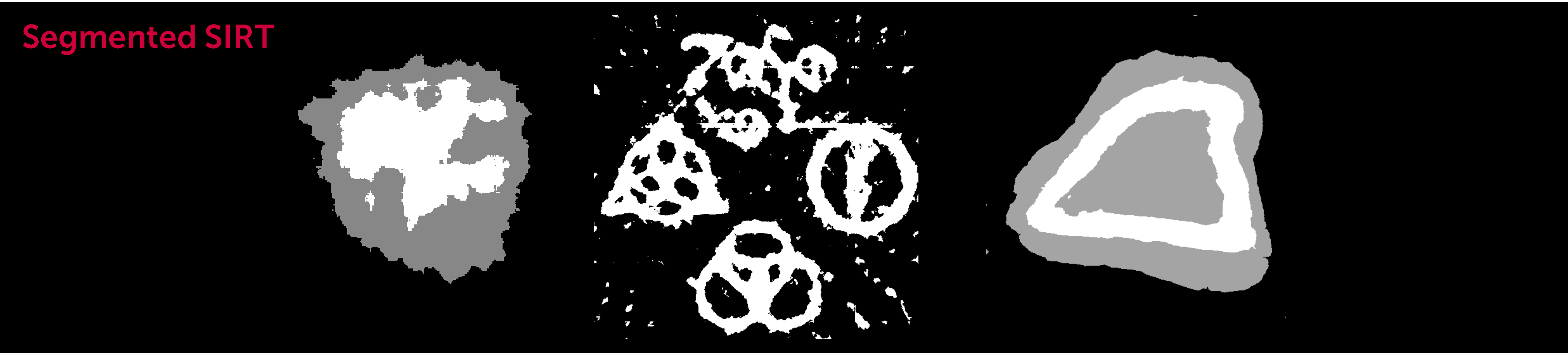

projection data
segment
threshold value: 128
grey levels: 0, 255
reconstruct
reconstruction
segmentation
fixed pixels
update pixels
project
+
-
residual projection data
reconstruct
limited to
update pixels
blur
reconstruction


iteration 1
error pixels: 9491
iteration 10
error pixels: 5460
iteration 50
error pixels: 233


15000
10000
5000
0
0
50
100
error pixels
iteration

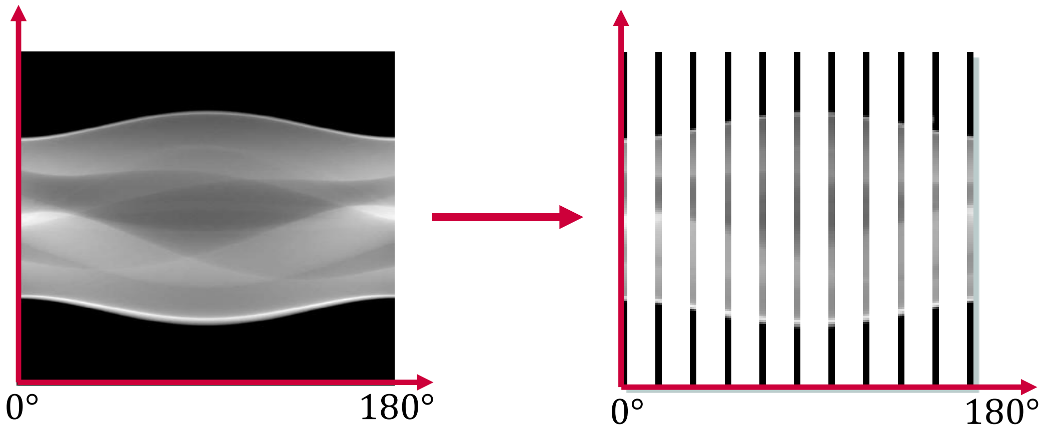
Small number of projections

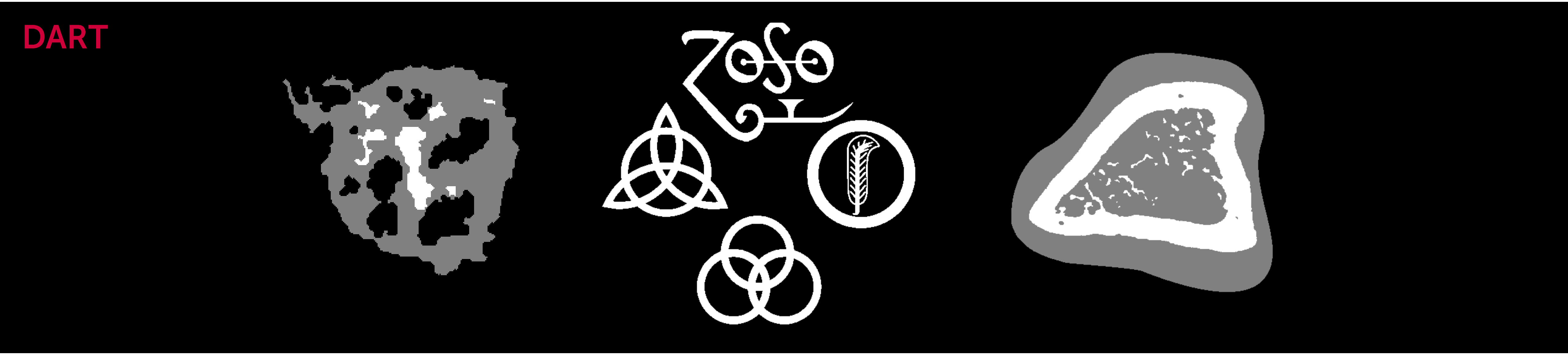




10 projections | 20 projections | 15 projections

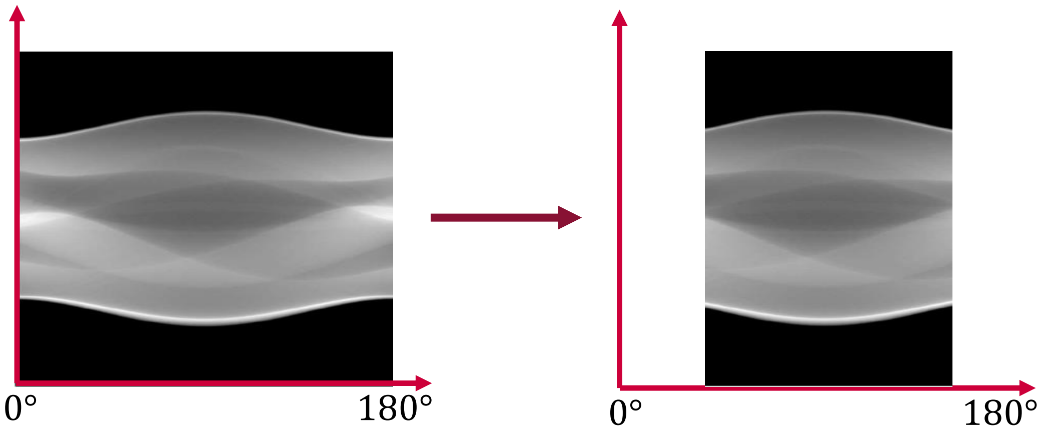
Small number of projections

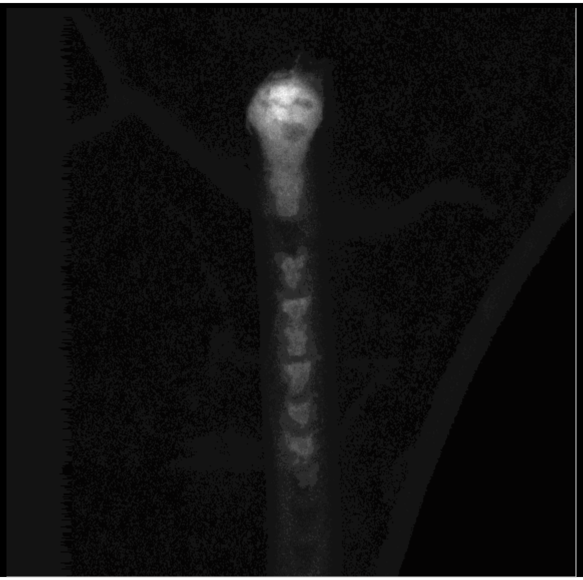


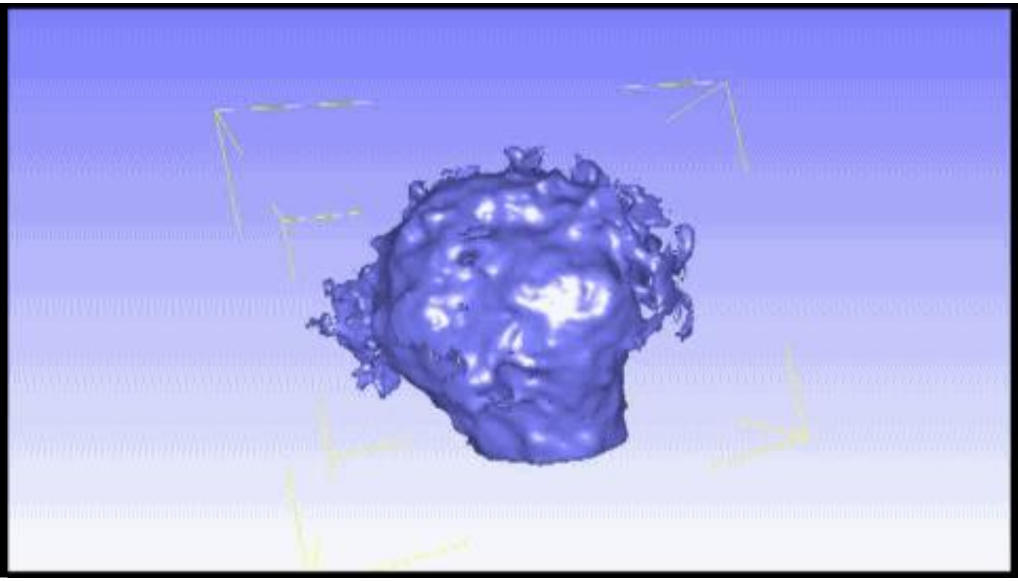


10 projections   20 projections   15 projections

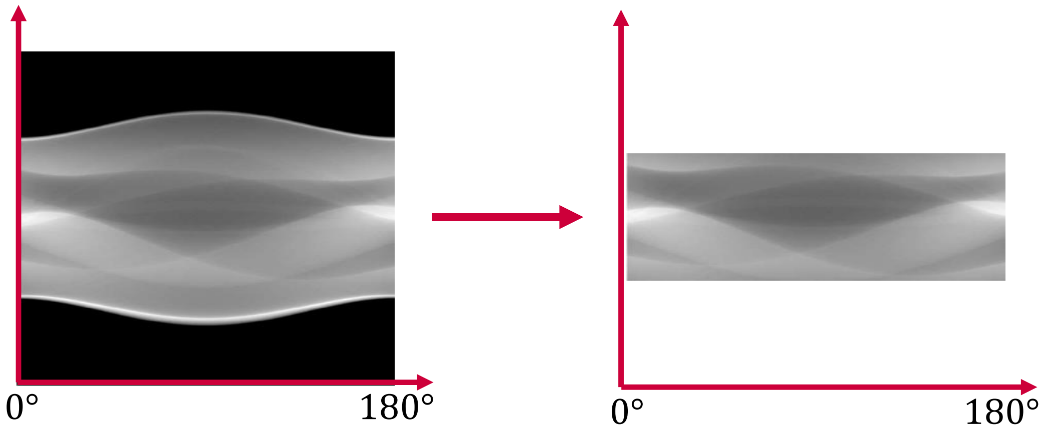
Missing wedge

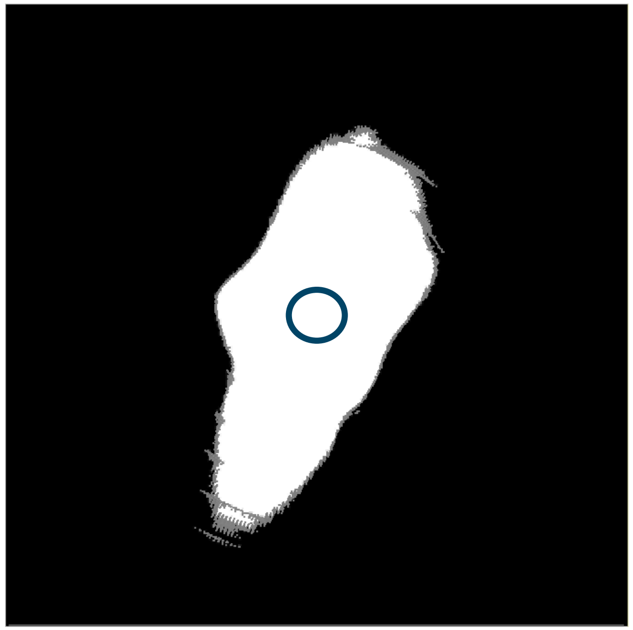






Truncated projection





Overview

Discrete Tomography

- Causes of insufficient data
- DART
- Some examples

Reality check

- Prior knowledge
- Material imperfections
- Partial volume effect
- Projection noise
- Beam hardening

Improving DART

- Grey level estimation
- softDART
- Spatial Coherence Prior

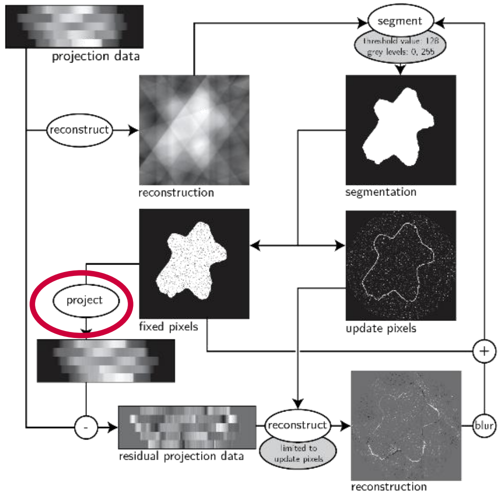
Ideally...

- Attenuation exactly known
- Materials homogeneous
- Large objects
- No/little noise
- Monochromatic X-ray beam

Reality...

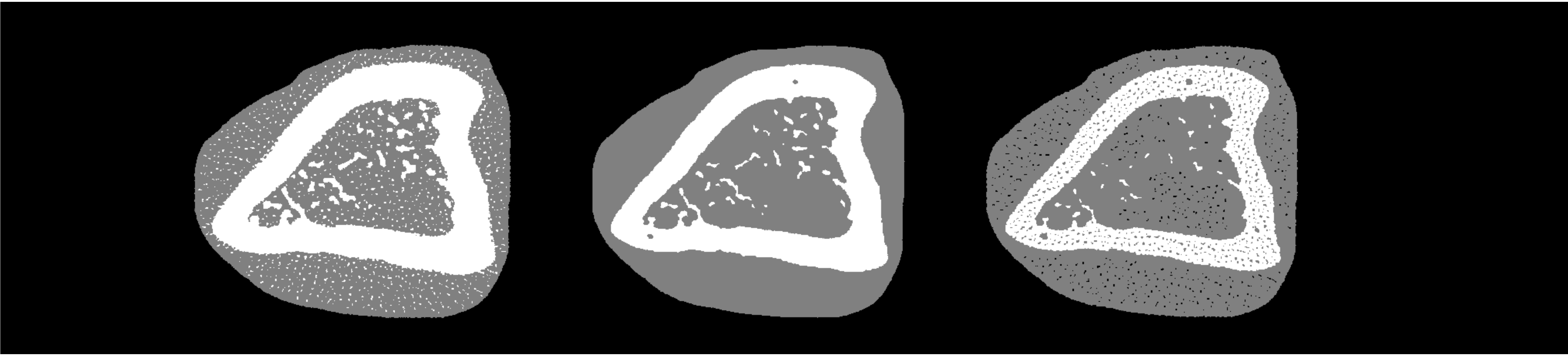
- Unaccurate prior knowledge
- Density perturbations
- Small structures
  - Partial volume effects
- Noisy data
  - Metal artefacts
- Polychromatic X-ray beam
  - Beam hardening

Attenuation values



Attenuation coefficients should be known

- Too low: residual norm too high for reconstruction mask
- Too high: residual norm too low for reconstruction mask



Correct values - 10%
rmse: 0. 1706

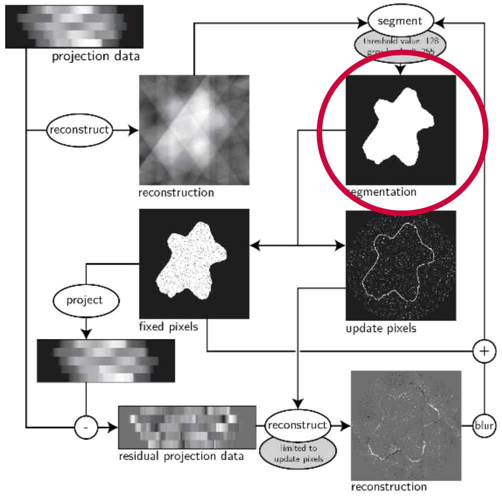
Correct values
rmse: 0.0183

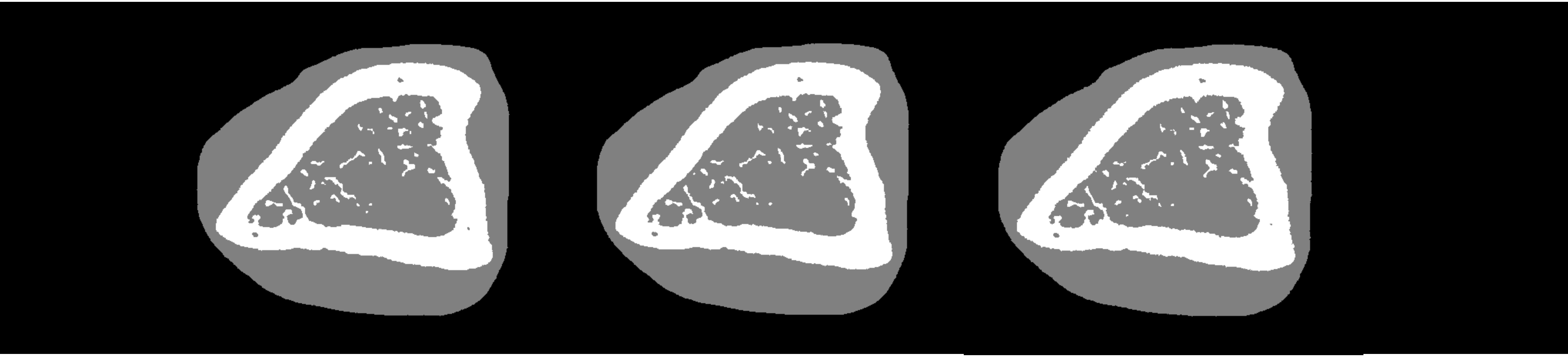
Correct values + 10%
rmse: 0. 1477

Material imperfections

Sometimes a homogeneous material is not really homogeneous

- Small perturbations in the density





No perturbation
rmse: 0. 0188

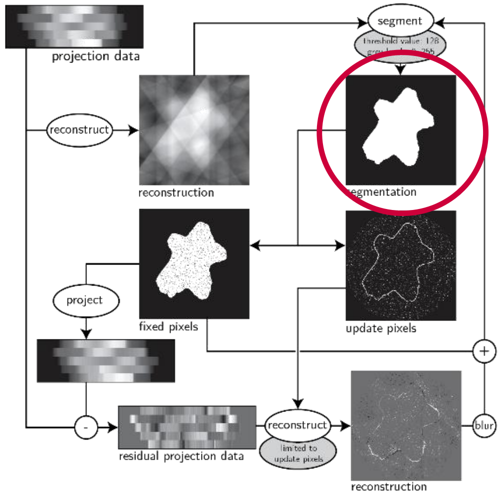
Perturbation: $\sigma = 5\%$
rmse: 0.0206

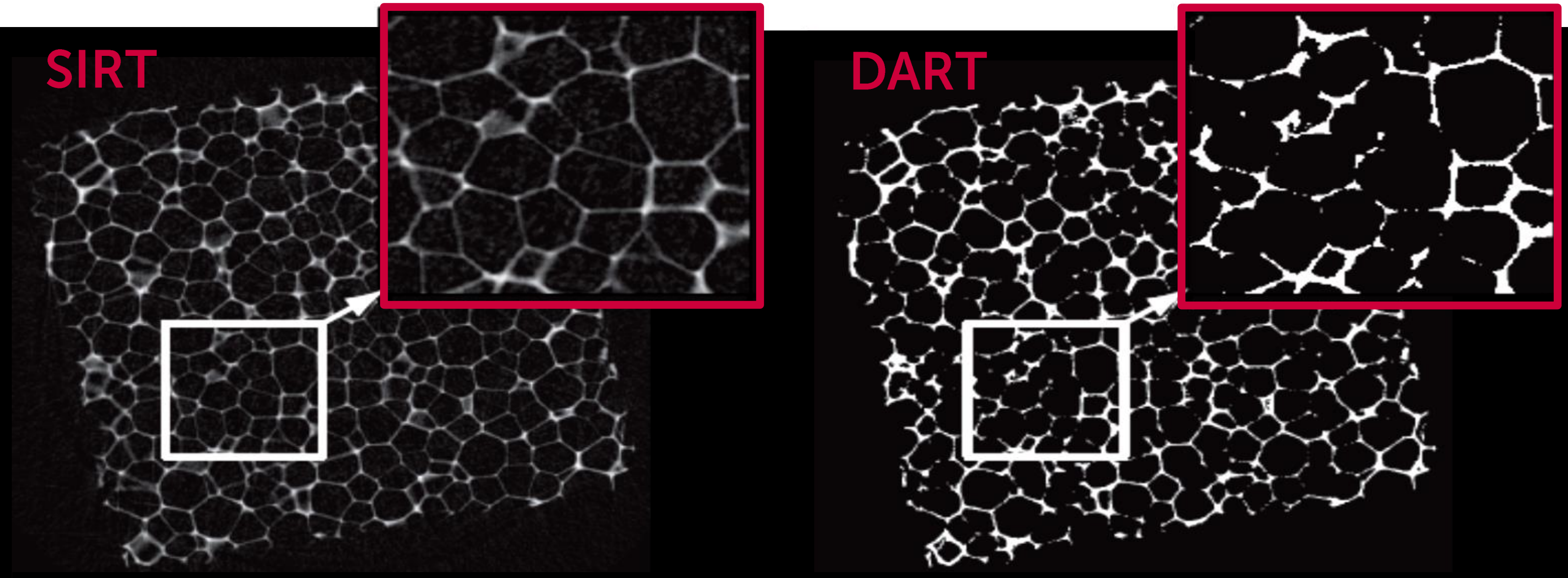
Perturbation: $\sigma = 10\%$
rmse: 0. 0579

Partial Volume Effect

DART likes large objects

- Pixels must contain either material 1 or material 2, can't contain both
- What at the edge of an object?

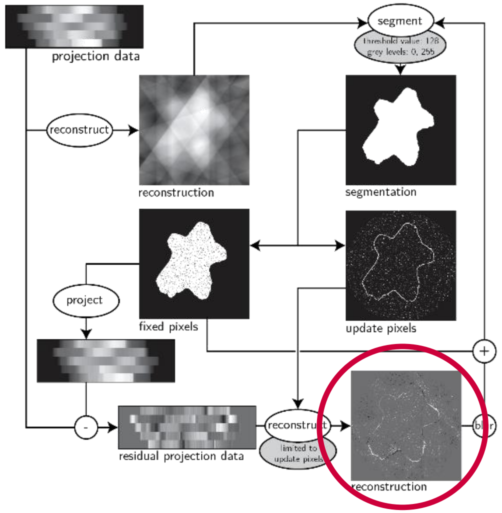


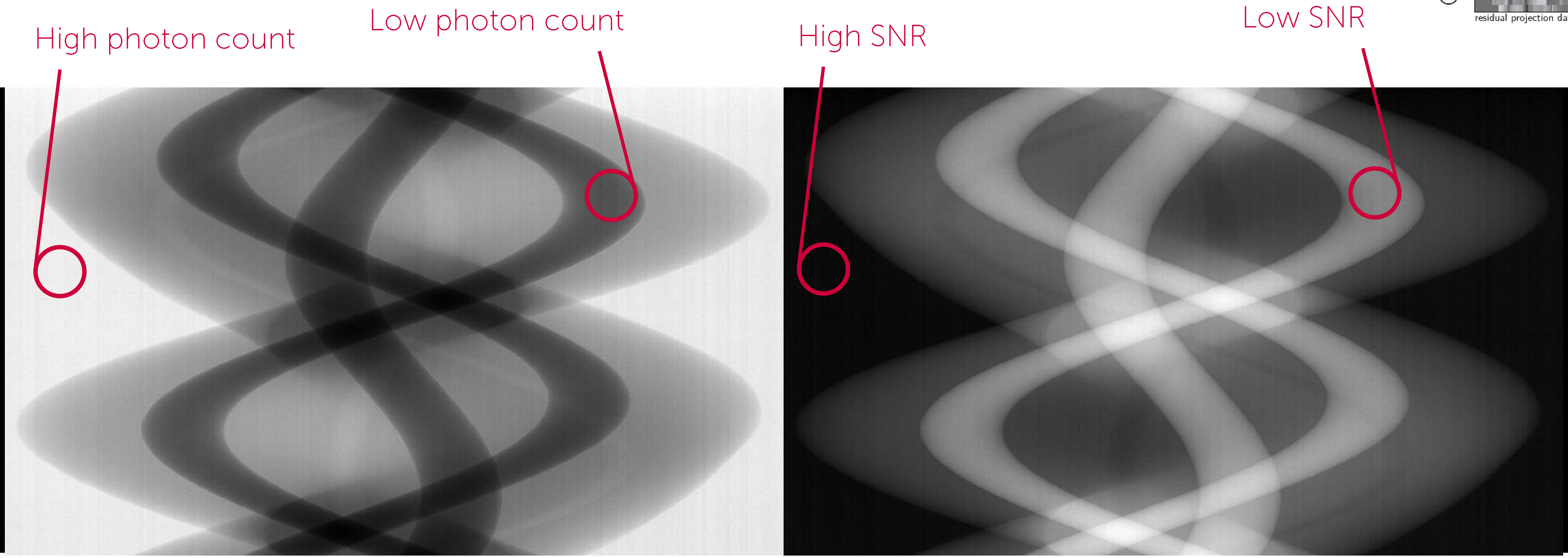


Noisy data

Projection data contains noise

- Poisson noise: SNR depends on the measured intensity (Beer Lambert law)
- SNR of residual projection decreases while #reconstruction pixels also decreases





Intensity values I
measured

Attenuation values
used in reconstruction

$$p = -\log\left(\frac{I}{I_0}\right)$$

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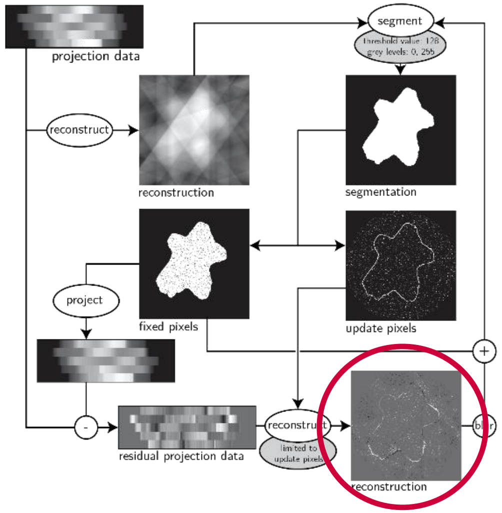
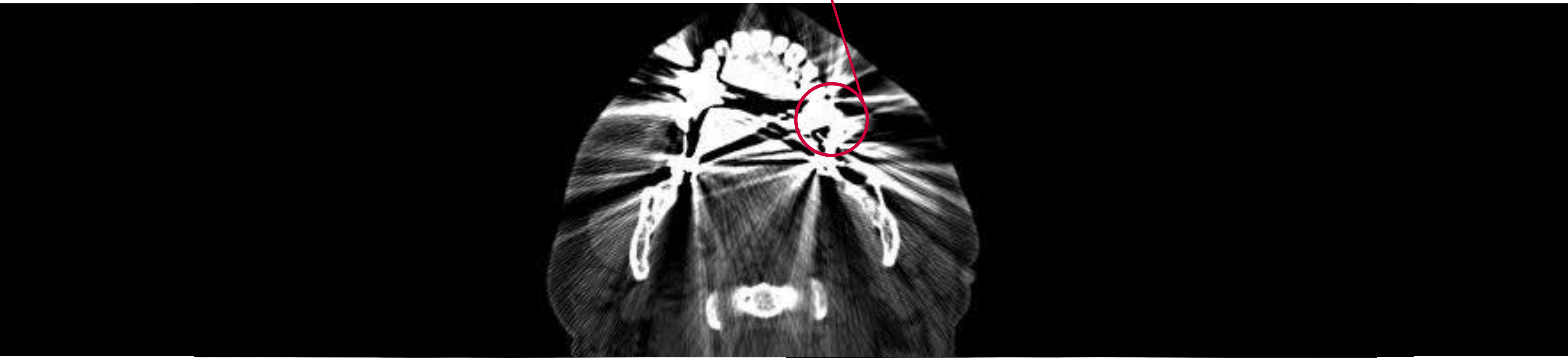


Metal artefacts



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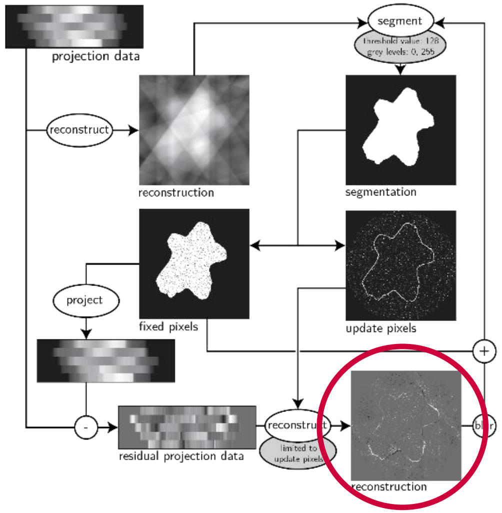
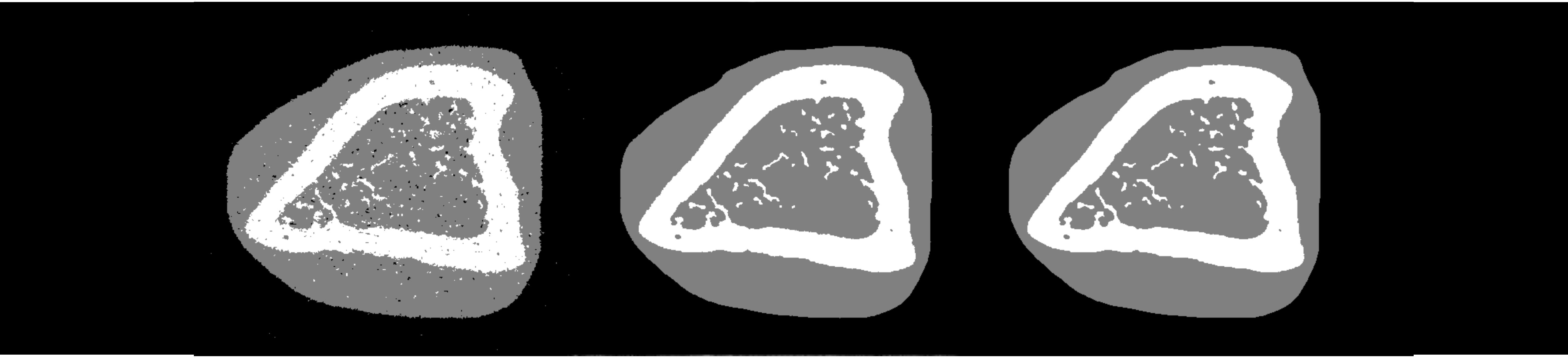




Loads of noise
rmse: 0. 0487

Little bit of noise
rmse: 0.0191

No noise
rmse: 0. 0158

Polychromatic X-ray beam

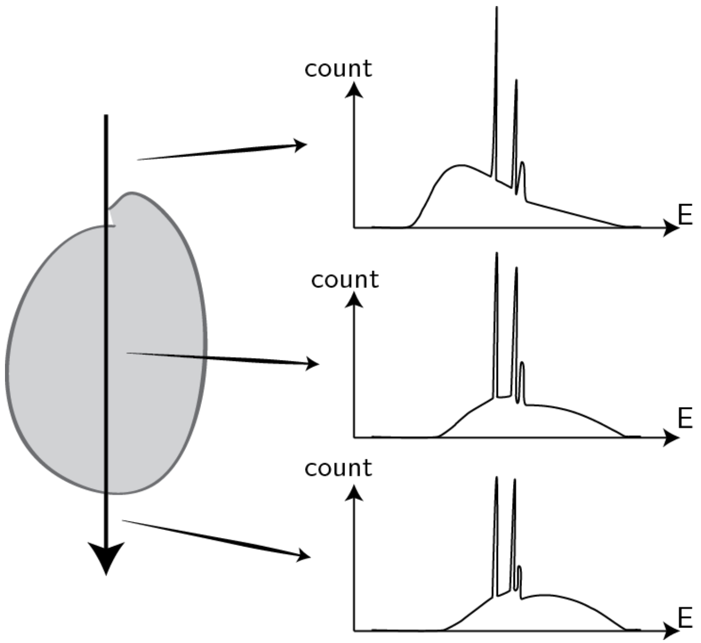
X-ray beams can be monochromatic

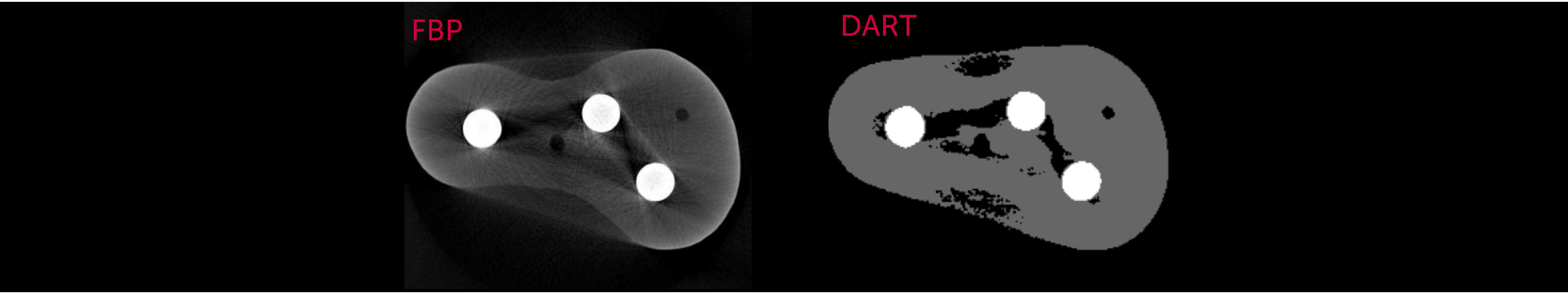
$$I^{mono}(\mu) = I_0\, e^{-\sum_m t_{hm}\mu_m}$$

but are usually polychromatic

$$I^{poly}(\mu) = \int I_{E,0}\, e^{-\sum_m t_{hm}\mu_{E,m}}\, dE$$

which leads to beam hardening and beam hardening artefacts





courtesy of Gert Van Gompel

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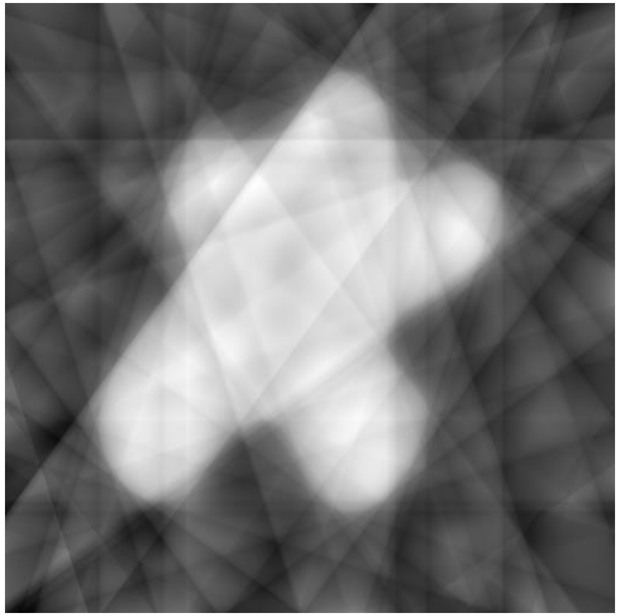
Grey level estimation

Option 1: Manually

- Based on classic reconstruction
- Using trial-and-error



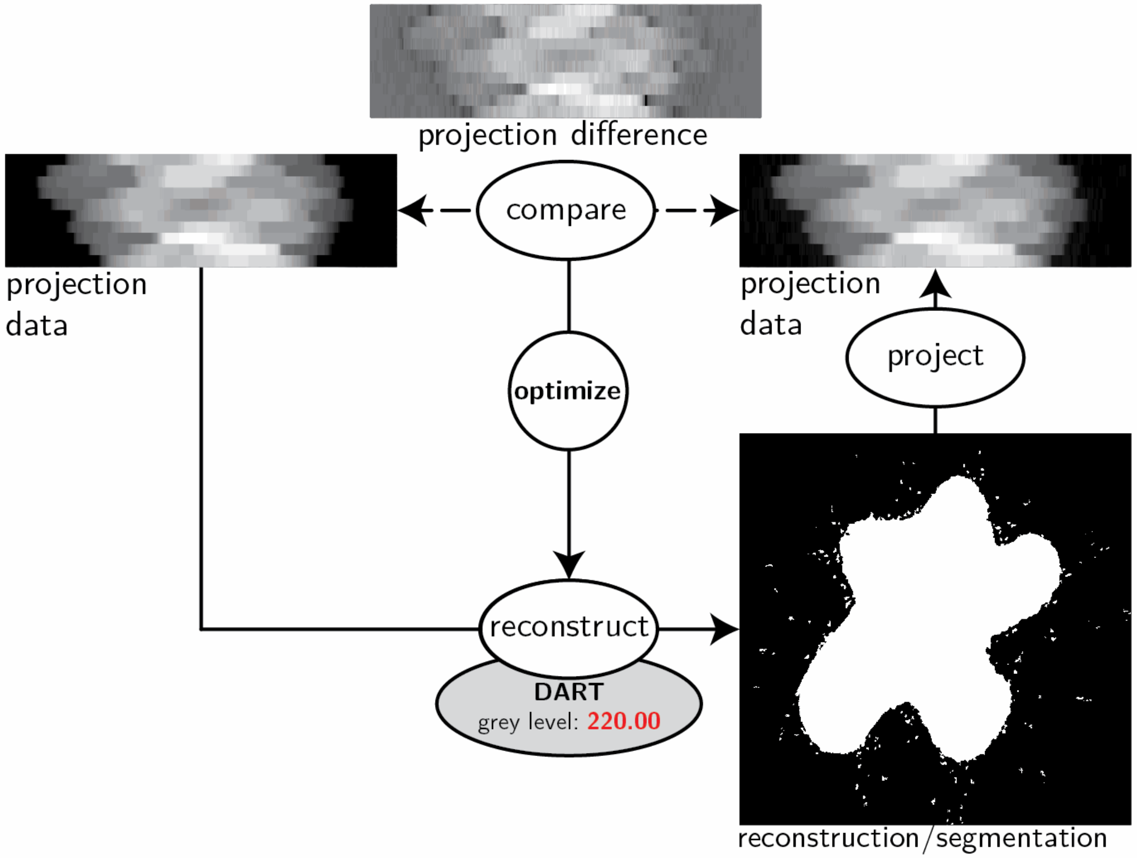
Grey level estimation

Option 2: External Optimization Strategy

Optimal grey level: DART reconstruction adheres maximally to the projection data

Optimization:

- Simplex search
- Pattern search
- Adaptive surrogate modelling
- …

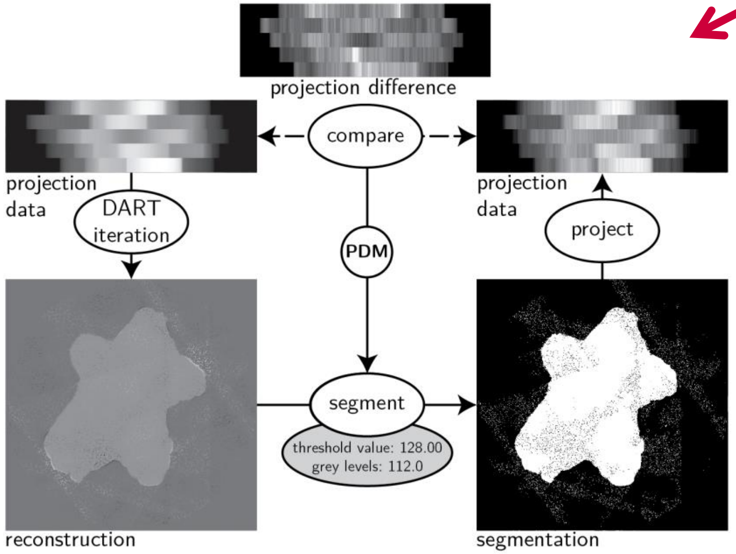


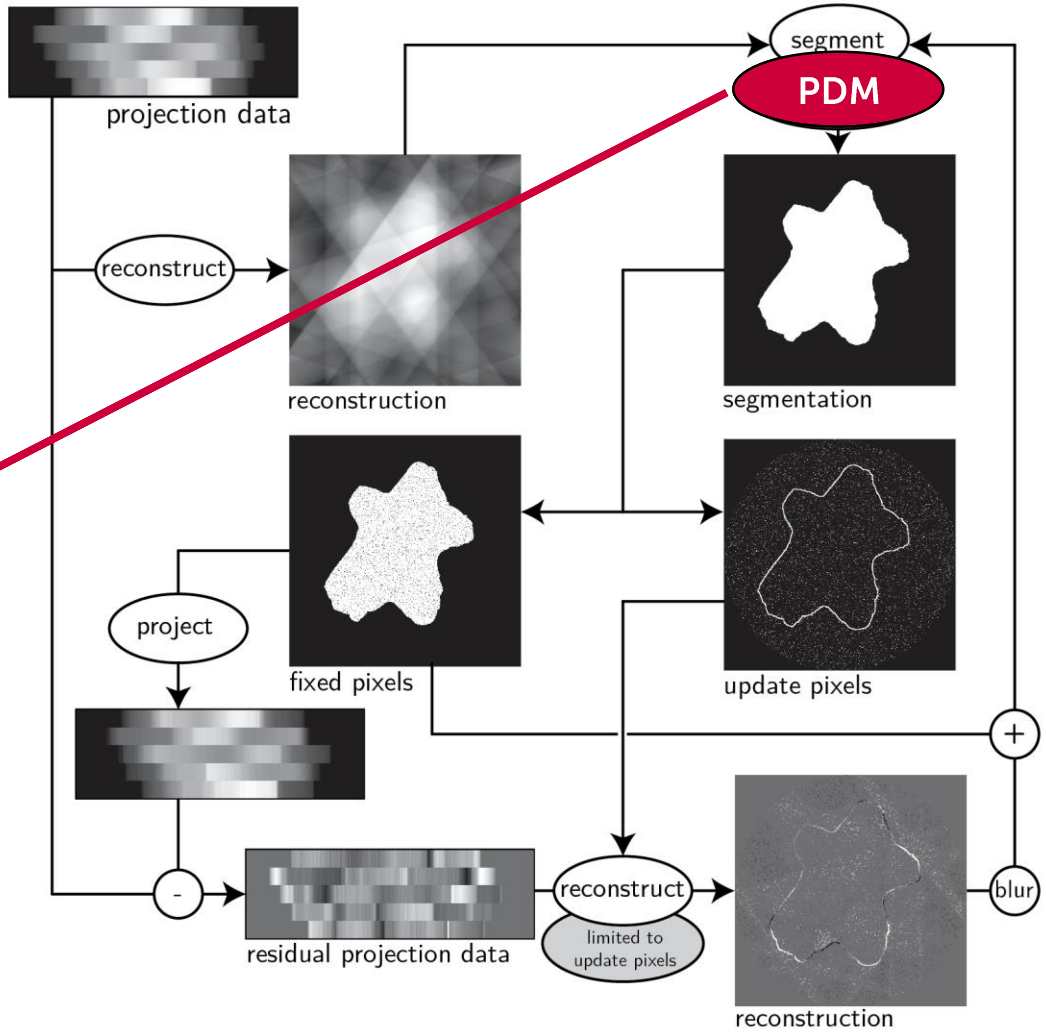
Grey level estimation

Option 3: Estimation during DART

Alternate

- DART iteration
- Grey level estimation with Projection Distance Minimization (PDM)





DART limitations

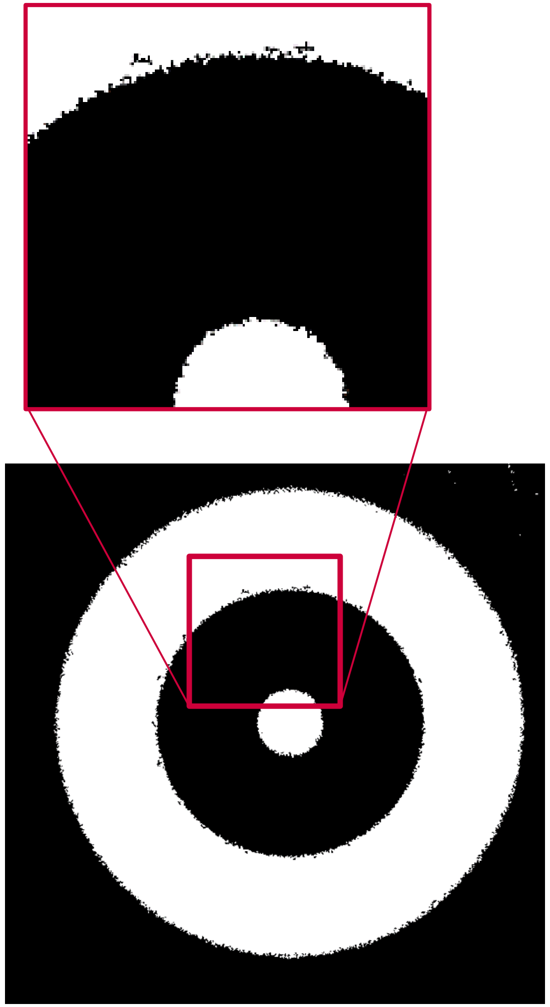
Hard constraints on attenuation coefficients are vulnerable to

- Regions with non-uniform attenuation
- Noise in projection data

Poor shape prior

- Only deals with “smooth” boundaries
- Does not favor one segmentation to another

Example



DART
20 projections

softDART

DART: binary reconstruction mask

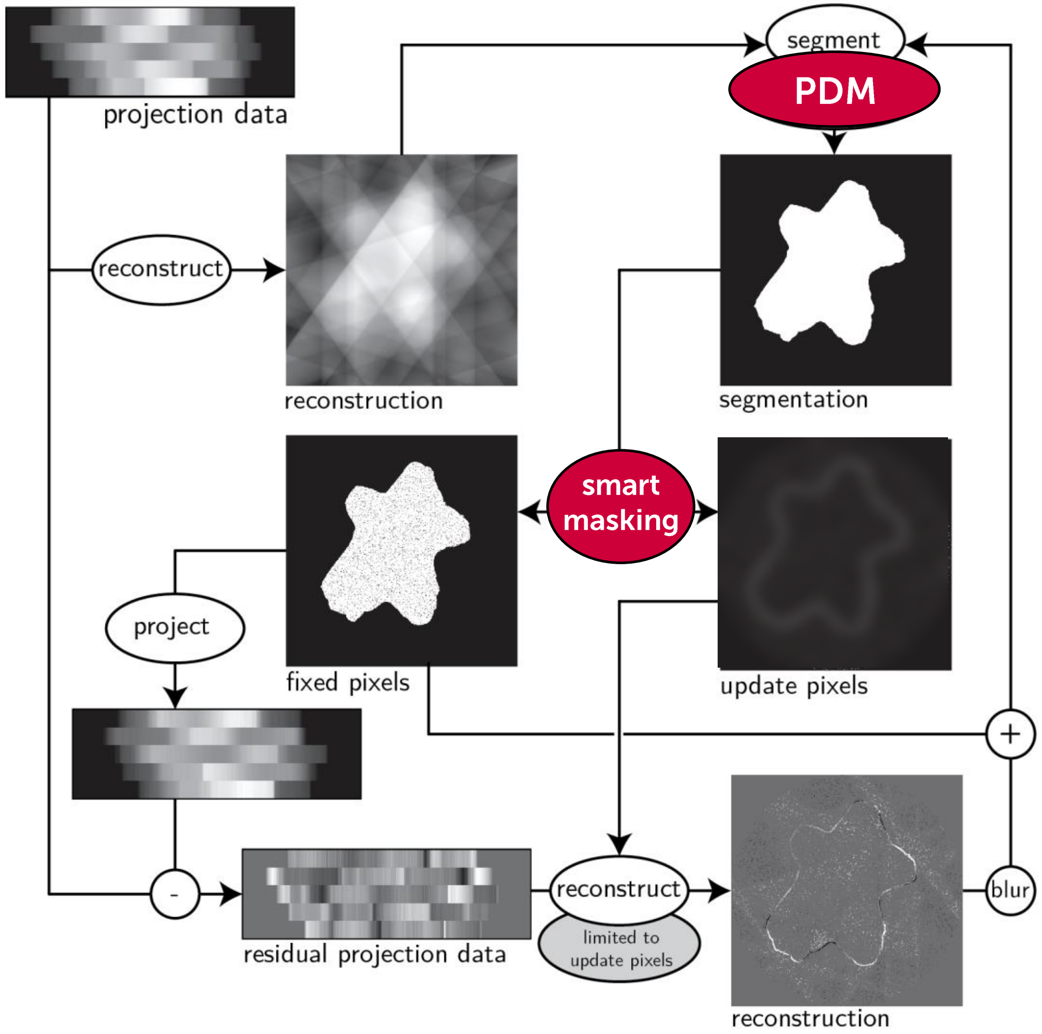
solve $W_M v_M = p - W_{\bar{M}} s$

softDART: smart reconstruction mask

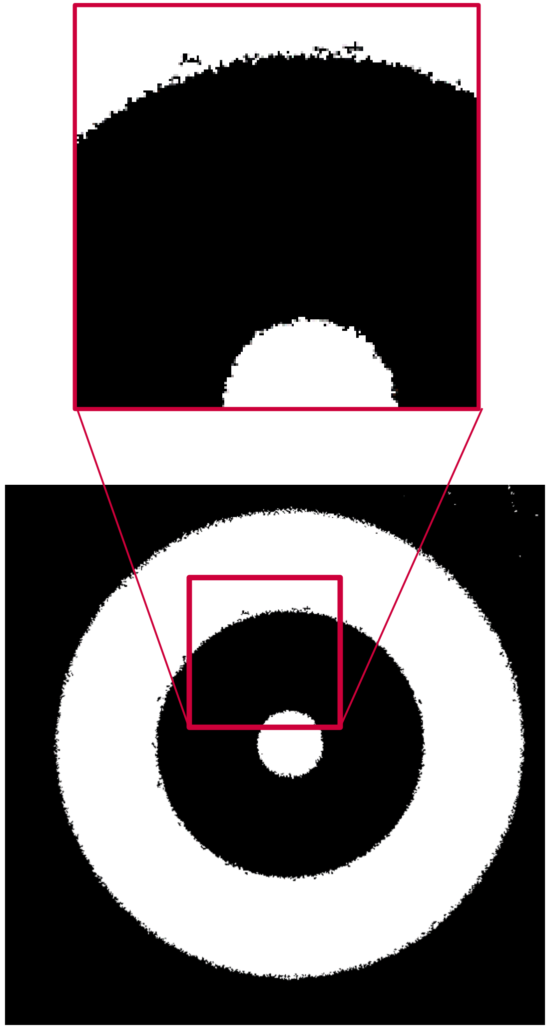
solve $WMv = p - W(I - M)s$

$M \in \mathbb{R}^{n \times n}$ is diagonal uncertainty matrix

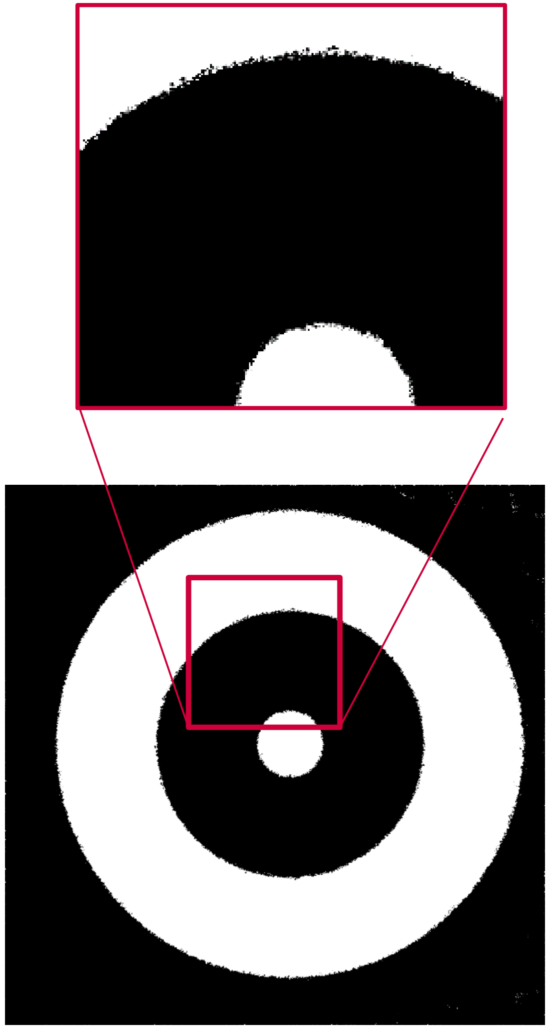
- distance from edge
- statistics from previous iterations
- prior knowledge
- ...



Example



DART
20 projections



softDART
20 projections

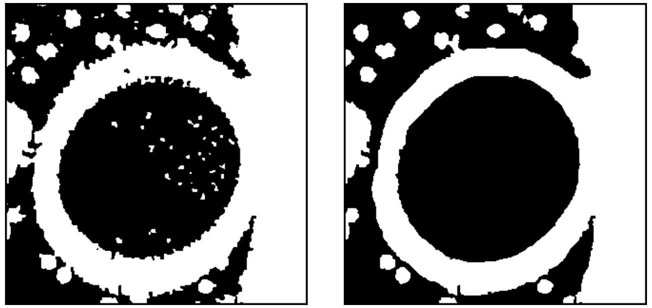
Spatial Coherence Prior

Same value pixels stick together

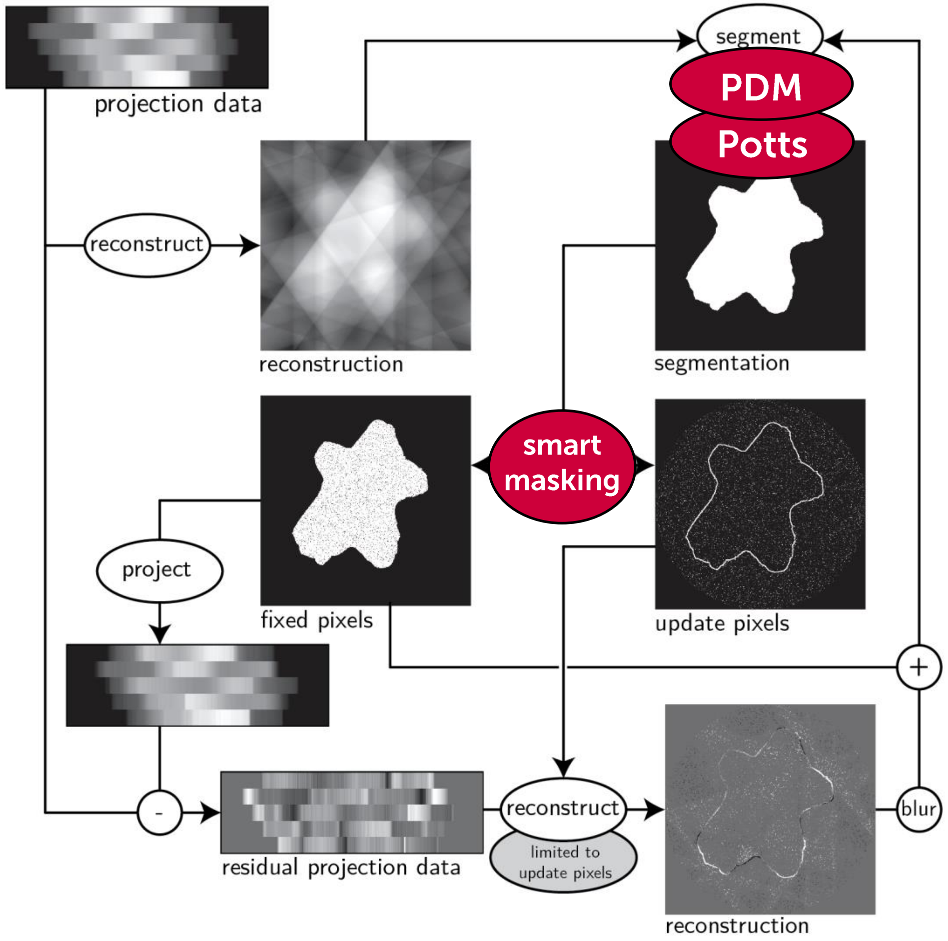
Potts Prior model

Likelihood of s being the correct segmentation, given that it is likely to be spatially coherent

$$p(s \mid J) = \frac{1}{Z(J)} \exp\left(J \sum_{i} \sum_{i' \in \varkappa(i)} \delta(s_i, s_{i'}) \right)$$



exp(-1.3e5) < exp(-9.6e4)

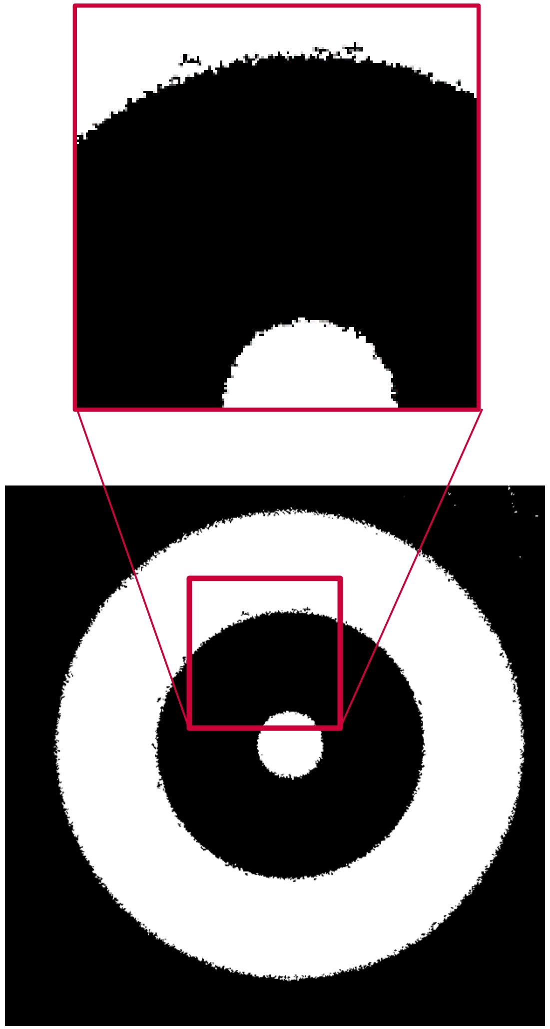


Precise optimization is NP-hard

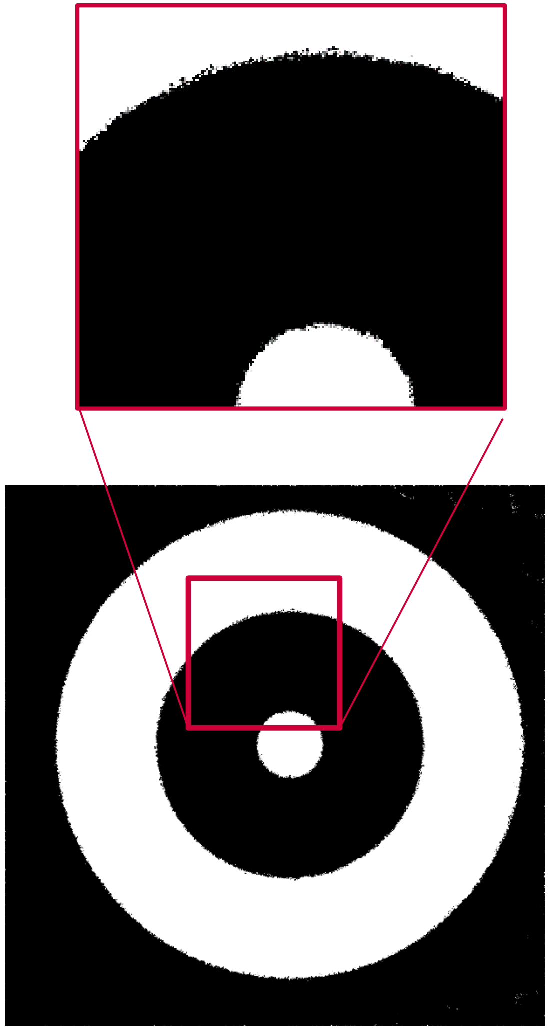
We used Markov Chain Monte Carlo (MCMC)

- Gibbs sampler
- Simulated annealing

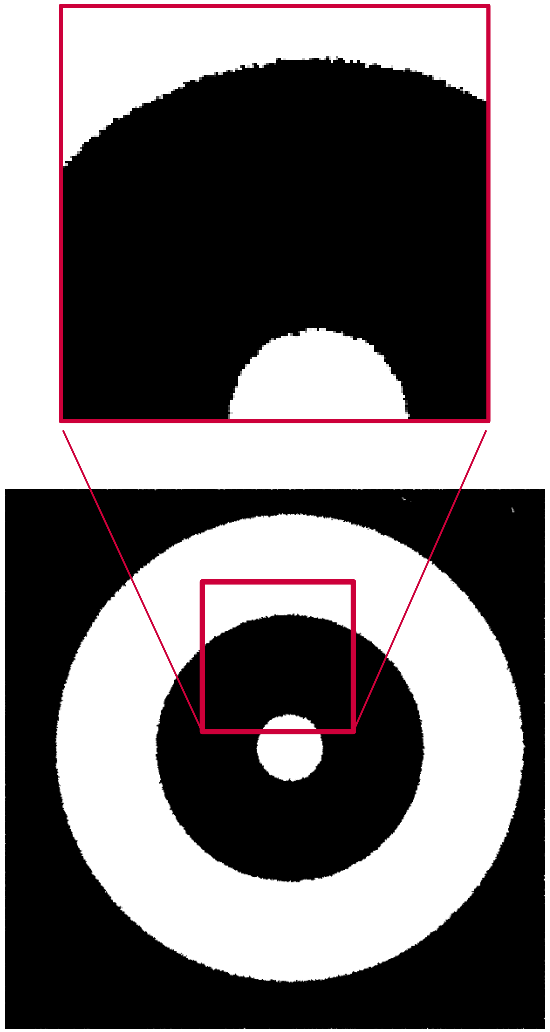
Example



DART
20 projections

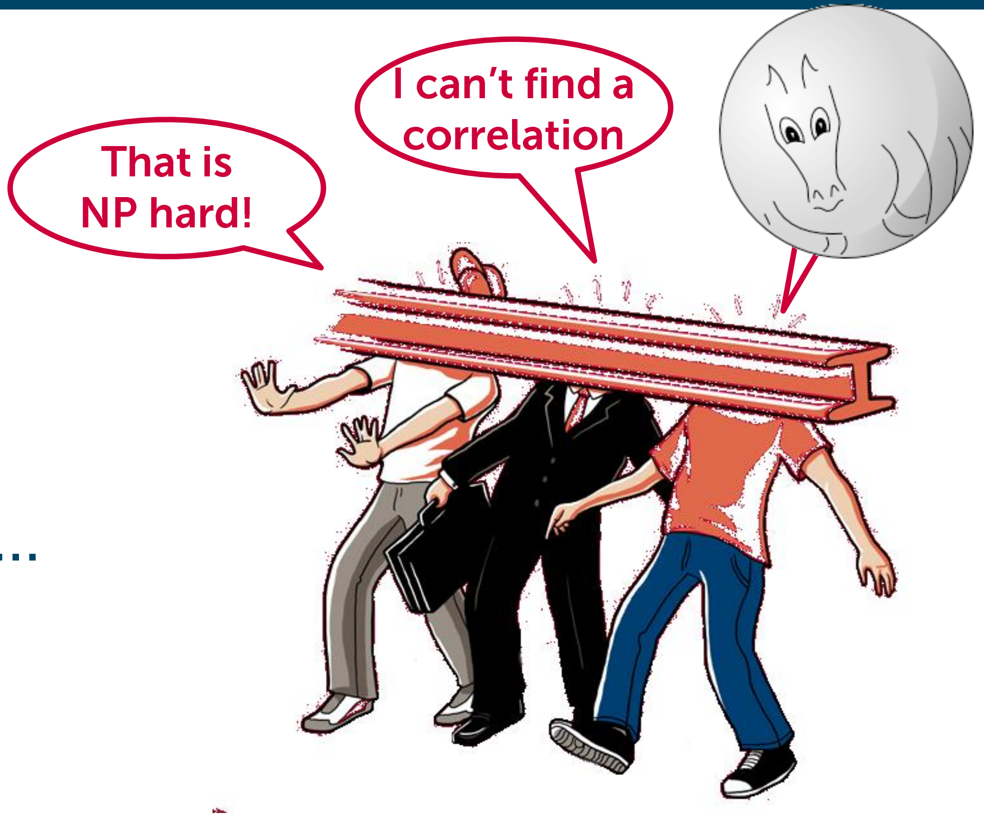


softDART
20 projections



softDART + Potts prior
20 projections

Conclusions



Assumptions and prior knowledge are necessary…

…but don't rely on them too much.

- Try to replace or test prior knowledge with objective functions
- Prefer soft constraints over hard constraints