Recent advances in filter based tomographic reconstruction methods

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Introduction

- Modern advanced tomographic experiments produce increasingly limited data
  - 4D tomography, in-vivo tomography, ...

- Advanced reconstruction methods can produce **accurate reconstructions** from limited data

- Observation: advanced methods are **rarely used** in practice
Problems with advanced methods

- Observation: advanced methods are rarely used in practice
- Several causes:
  - Computation time
  - Unknown parameters
  - Difficult practical implementation
- Filtered backprojection is very efficient and easy to use
  - Good implementations often available at experimental facilities
- Idea: improve FBP to resemble advanced methods
Filtered backprojection

- FBP first convolves the projection data with a filter $h$, then backprojects the result:

$$FBP(p, h) = W^T C_h p$$

- Usually a standard filter is used (e.g. the ram-lak filter):

- Idea: Change the filter of FBP to approximate slower methods
Three approaches

- Idea: *Change* the filter of FBP to approximate slower methods
- This talk: Discuss *three* recent approaches

- Use a filter that *depends on the data*
- Use a filter that approximates an algebraic method
- Use a filter that is trained by neural networks

Approach #1: Data-dependent filter

- Many algebraic methods find an image that minimizes projection error:

\[ \mathbf{x}_{alg} = \underset{x}{\text{argmin}} \| \mathbf{p} - W \mathbf{x} \|_2 \]

- Idea: find a filter such that the resulting FBP reconstruction minimizes projection error:

\[ \mathbf{h}^* = \underset{h}{\text{argmin}} \| \mathbf{p} - W \text{FBP}(\mathbf{p}, h) \|_2 \]

- Advantage: a much smaller linear system
  - Faster to solve
Approach #1: Data-dependent filter

1024 x 1024 pixels, 64 projections, Poisson noise

(a) FBP  (b) SIRT  (c) MR-FBP
Approach #1: Data-dependent filter

- Prior knowledge can be added to improve quality

1024 x 1024 pixels, 64 projections, Poisson noise

(a) FBP  (b) SIRT  (c) MR-FBP + prior
Three approaches

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Approach #2: Approximate algebraic method

- Take the standard equation for the algebraic SIRT method:

\[ x^{i+1} = x^i + \alpha W^T (p - W x^i) \]

- We can rewrite this in matrix form:

\[ x^{i+1} = (I - \alpha W^T W) x^i + \alpha W^T p \]

- This is a recurrence relation, with solution for iteration \( n \):

\[ x^n = A^n x^0 + \alpha \left[ \sum_{k=0}^{n-1} A^k \right] W^T p , \quad A = (I - \alpha W^T W) \]
Approach #2: Approximate algebraic method

- We have rewritten the SIRT equation to:

\[ x^n = \alpha \left[ \sum_{k=0}^{n-1} A^k \right] W^T p \]

- Compare with the “backproject, then filter” FBP equation:

\[ FBP(p, h') = C_{h'} W^T p \]

- Approximate the \( A^k \) sum with a convolution
  - Filter can be precalculated for a certain acquisition geometry

- Computation time of reconstruction is identical to FBP
Approach #2: Approximate algebraic method

1024 x 1024 pixels, 256 projections, Poisson noise

(a) FBP
(b) SIRT
(c) SIRT-FBP
Approach #2: Approximate algebraic method

Comparison of algebraic approximation filters

- Ram-Lak filter
- $N_\theta = 64$, 200 iterations
- $N_\theta = 64$, 1000 iterations
- $N_\theta = 256$, 200 iterations
- $N_\theta = 256$, 1000 iterations

Filter amplitude vs. Frequency
Three approaches

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Approach #3: Neural networks

Diagram:

- **Sinogram**
  - Connections: $f_0$, $f_1$, $f_2$, $f_3$
  - Outputs to:
    - $f_0$ to $FBP$
    - $f_1$ to $FBP$
    - $f_2$ to $FBP$
    - $f_3$ to $FBP$

- **FBP**
  - Connections: $w_0$, $w_1$, $w_2$, $w_3$
  - Input from:
    - $f_0$
    - $f_1$
    - $f_2$
    - $f_3$

- **Reco**
  - Connections: $w_0$, $w_1$, $w_2$, $w_3$
  - Input from:
    - $f_0$
    - $f_1$
    - $f_2$
    - $f_3$
Approach #3: Neural networks

- Filters and weights are trained using neural network theory.
- To train, high-quality reconstructions of objects are needed:
  - Scan representative objects with high dose
  - Scan at the start/end of a dynamic experiment
  - ...
- The network will learn filters that exploit:
  - Acquisition details (noise profile, # projections, ...)
  - Object characteristics
- After training, reconstruction is fast and accurate
Approach #3: Neural networks
Approach #3: Neural networks

4k x 4k pixels, synchrotron data (ESRF)

(a) FBP (all projections)  (b) FBP (5%)  (c) NN-FBP (5%)
Approach #3: Neural networks
Conclusions

- FBP with non-standard filters can produce very accurate reconstructions

- The filter can be chosen in different ways, each with advantages and disadvantages

- MR-FBP
  - Use a data-dependent filter that minimizes the projection error

- SIRT-FBP
  - Use a filter that approximates an algebraic method

- NN-FBP
  - Train filters using high-quality training datasets
Thank you for listening!

For more information: D.M.Pelt@cwi.nl
Open source implementations available at: https://github.com/dmpelt/

References:


Approach #1: Data-dependent filter

Comparison of data-dependent filters

- PHANTOM1, $N_\theta = 64$
- PHANTOM1, $N_\theta = 64$, noise
- PHANTOM1, $N_\theta = 128$
- PHANTOM2, $N_\theta = 64$
- Ram-Lak filter
Approach #3: Neural networks

Trained filters