Recent advances in filter based tomographic reconstruction methods

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Centrum Wiskunde & Informatica

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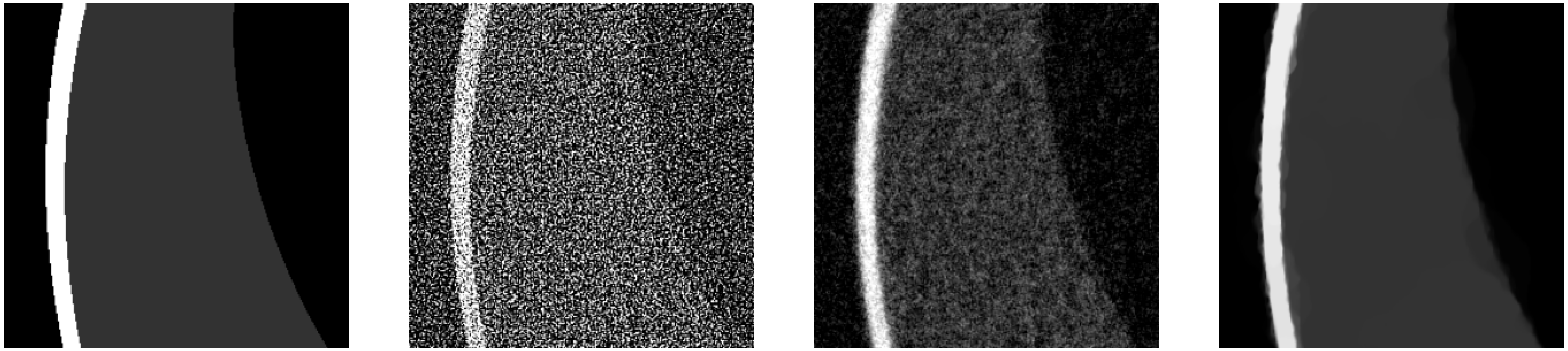
Introduction

- Modern advanced tomographic experiments produce increasingly limited data
  - 4D tomography, in-vivo tomography, ...
- Advanced reconstruction methods can produce accurate reconstructions from limited data



(a) Phantom (b) FBP (c) SIRT (d) TV-MIN

- Observation: advanced methods are rarely used in practice

Problems with advanced methods

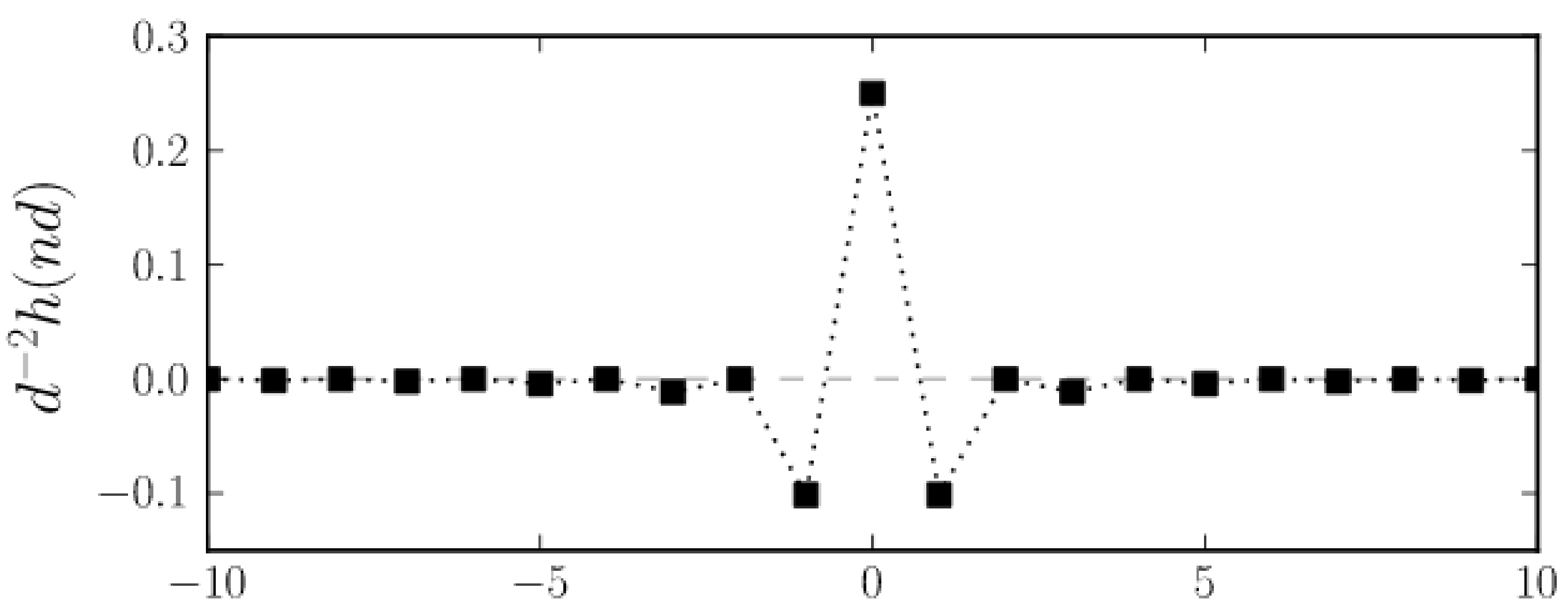
- Observation: advanced methods are rarely used in practice
- Several causes:
  - Computation time
  - Unknown parameters
  - Difficult practical implementation
- Filtered backprojection is very efficient and easy to use
  - Good implementations often available at experimental facilities
- Idea: improve FBP to resemble advanced methods

Filtered backprojection

- FBP first convolves the projection data with a filter h, then backprojects the result:

$$FBP(\boldsymbol{p}, \boldsymbol{h}) = \boldsymbol{W}^T \boldsymbol{C_h} \boldsymbol{p}$$

- Usually a standard filter is used (e.g. the ram-lak filter):



- Idea: Change the filter of FBP to approximate slower methods

Three approaches

- Idea: Change the filter of FBP to approximate slower methods
- This talk: Discuss three recent approaches

- Use a filter that depends on the data
- Use a filter that approximates an algebraic method
- Use a filter that is trained by neural networks

[1] Pelt, D. M., & Batenburg, K. J. (2014). Improving Filtered Backprojection Reconstruction by Data-Dependent Filtering. *Image Processing, IEEE Transactions on, 23*(11), pp. 4750-4762.

Approach #1: Data-dependent filter

- Many algebraic methods find an image that minimizes projection error:

$$\boldsymbol{x}_{alg} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \|\boldsymbol{p} - \boldsymbol{W}\boldsymbol{x}\|_2$$

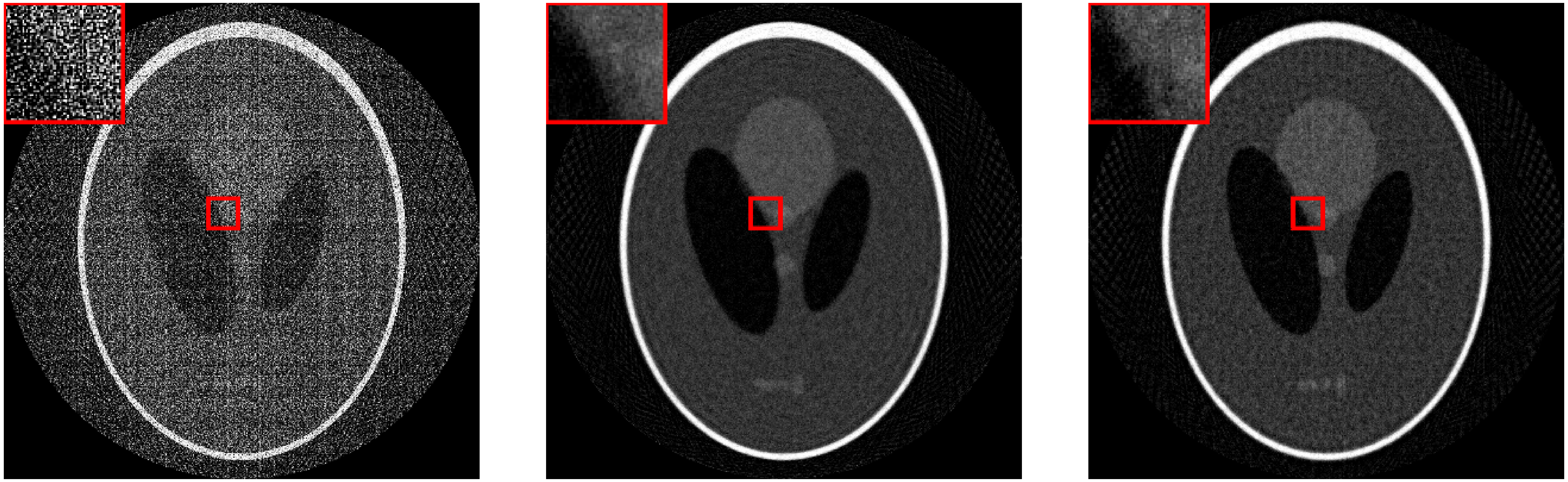
- Idea: find a filter such that the resulting FBP reconstruction minimizes projection error:

$$\boldsymbol{h}^* = \underset{\boldsymbol{h}}{\operatorname{argmin}} \|\boldsymbol{p} - \boldsymbol{W} FBP(\boldsymbol{p}, \boldsymbol{h})\|_2$$

- Advantage: a much smaller linear system
  - Faster to solve

Approach #1: Data-dependent filter

1024 x 1024 pixels, 64 projections, Poisson noise

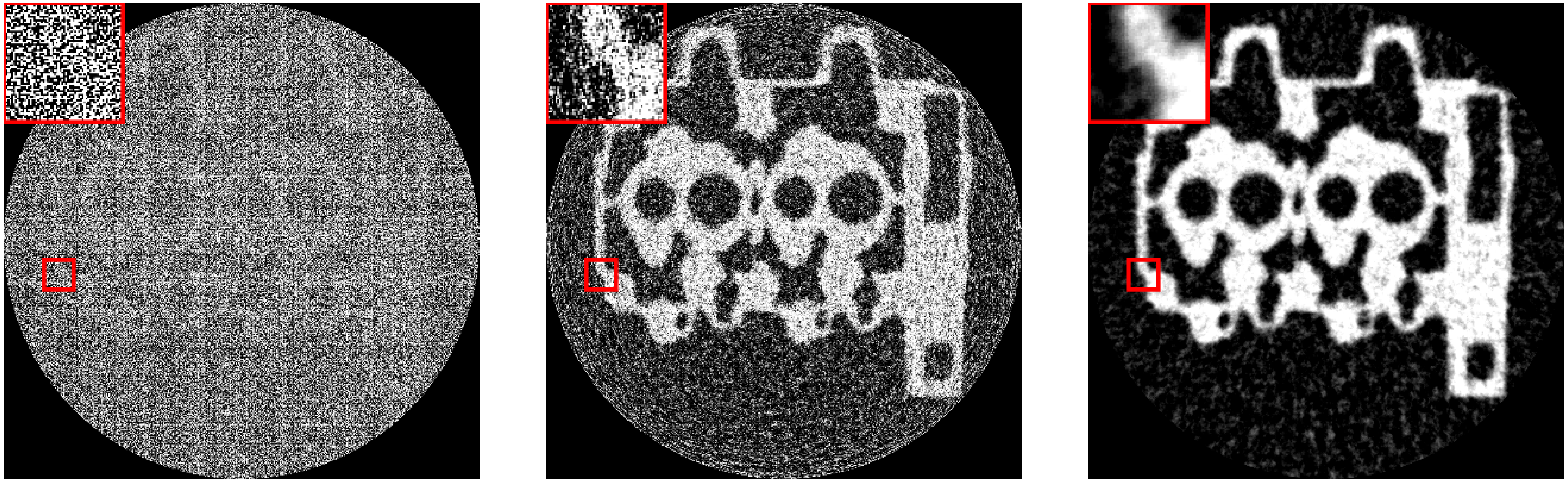


(a) FBP (b) SIRT (c) MR-FBP

Approach #1: Data-dependent filter

- Prior knowledge can be added to improve quality

1024 x 1024 pixels, 64 projections, Poisson noise



(a) FBP (b) SIRT (c) MR-FBP + prior

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Approach #2: Approximate algebraic method

- Take the standard equation for the algebraic SIRT method:

$$\boldsymbol{x}^{i+1} = \boldsymbol{x}^i + \alpha \boldsymbol{W}^T(\boldsymbol{p} - \boldsymbol{W}\boldsymbol{x}^i)$$

- We can rewrite this in matrix form:

$$\boldsymbol{x}^{i+1} = (\boldsymbol{I} - \alpha \boldsymbol{W}^T\boldsymbol{W})\boldsymbol{x}^i + \alpha \boldsymbol{W}^T\boldsymbol{p}$$

- This is a recurrence relation, with solution for iteration n:

$$\boldsymbol{x}^n = \boldsymbol{A}^n \boldsymbol{x}^0 + \alpha \left[\sum_{k=0}^{n-1} \boldsymbol{A}^k \right] \boldsymbol{W}^T\boldsymbol{p}, \quad \boldsymbol{A} = (\boldsymbol{I} - \alpha \boldsymbol{W}^T\boldsymbol{W})$$

Approach #2: Approximate algebraic method

- We have rewritten the SIRT equation to:

$$\boldsymbol{x}^n = \alpha \left[\sum_{k=0}^{n-1} \boldsymbol{A}^k \right] \boldsymbol{W}^T \boldsymbol{p}$$

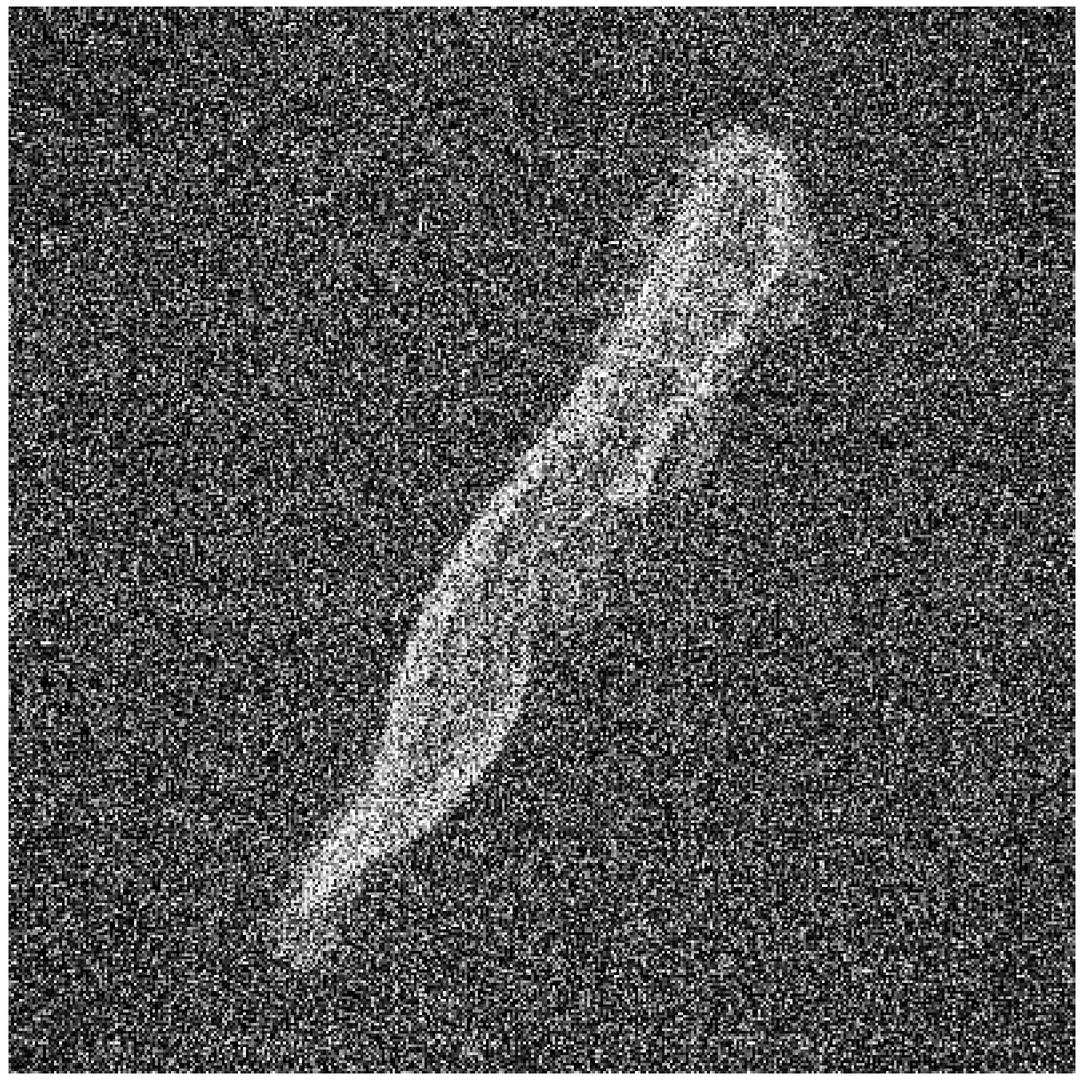
- Compare with the "backproject, then filter" FBP equation:

$$FBP(\boldsymbol{p}, \boldsymbol{h}') = \boldsymbol{C}_{\boldsymbol{h}'} \boldsymbol{W}^T \boldsymbol{p}$$

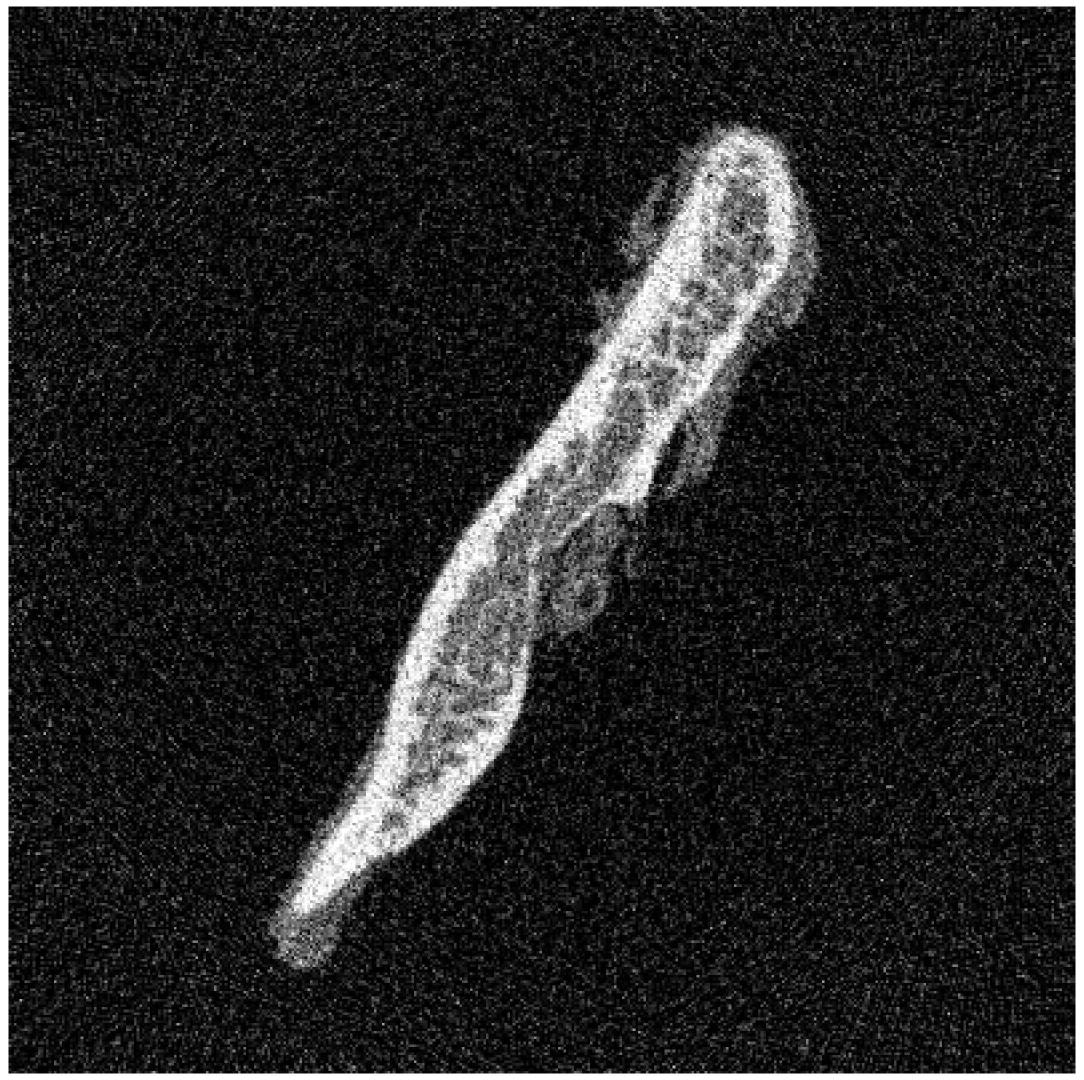
- Approximate the $\boldsymbol{A}^k$ sum with a convolution
  - Filter can be precalculated for a certain acquisition geometry
- Computation time of reconstruction is identical to FBP

Approach #2: Approximate algebraic method

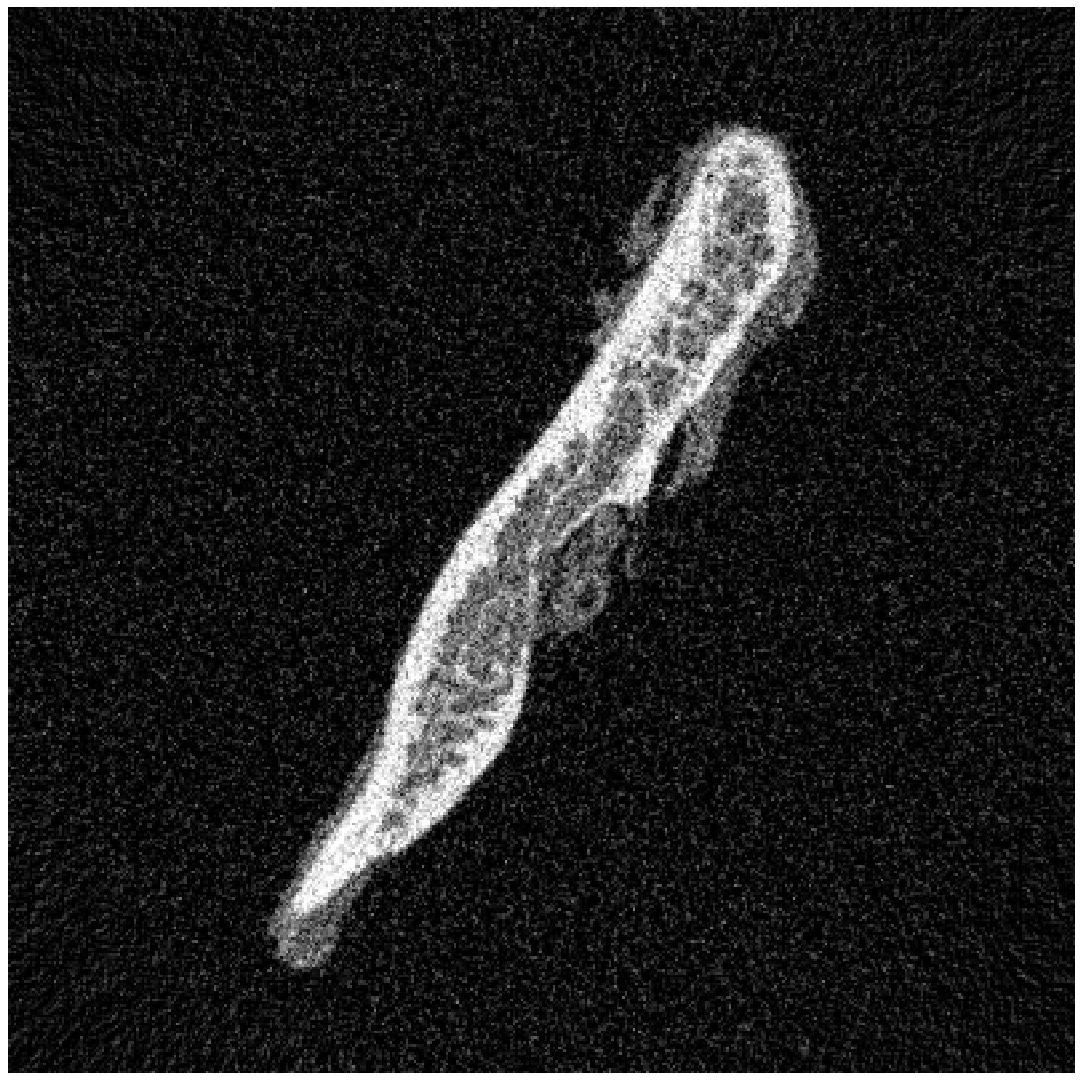
1024 x 1024 pixels, 256 projections, Poisson noise



(a) FBP



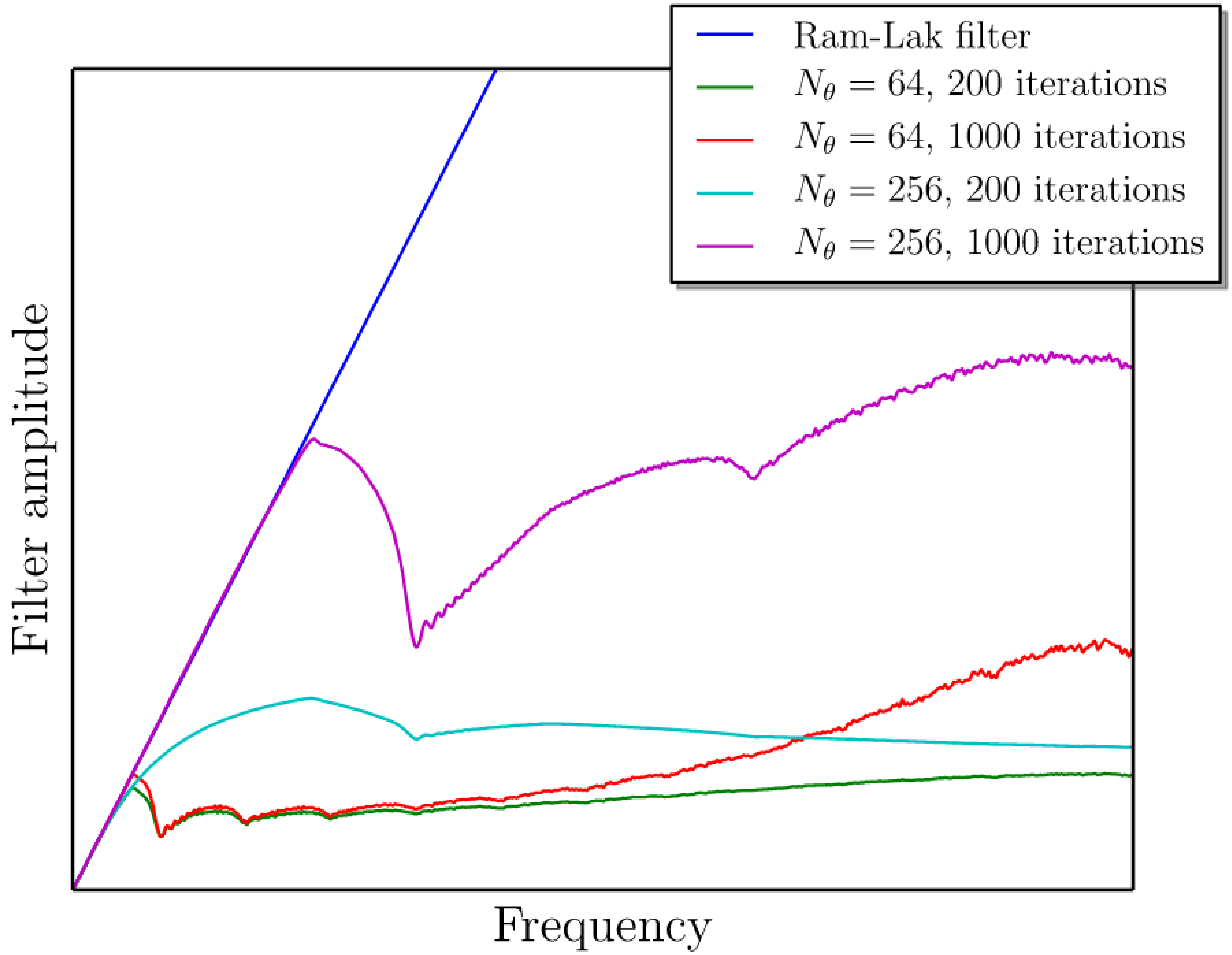
(b) SIRT



(c) SIRT-FBP

Approach #2: Approximate algebraic method

Comparison of algebraic approximation filters



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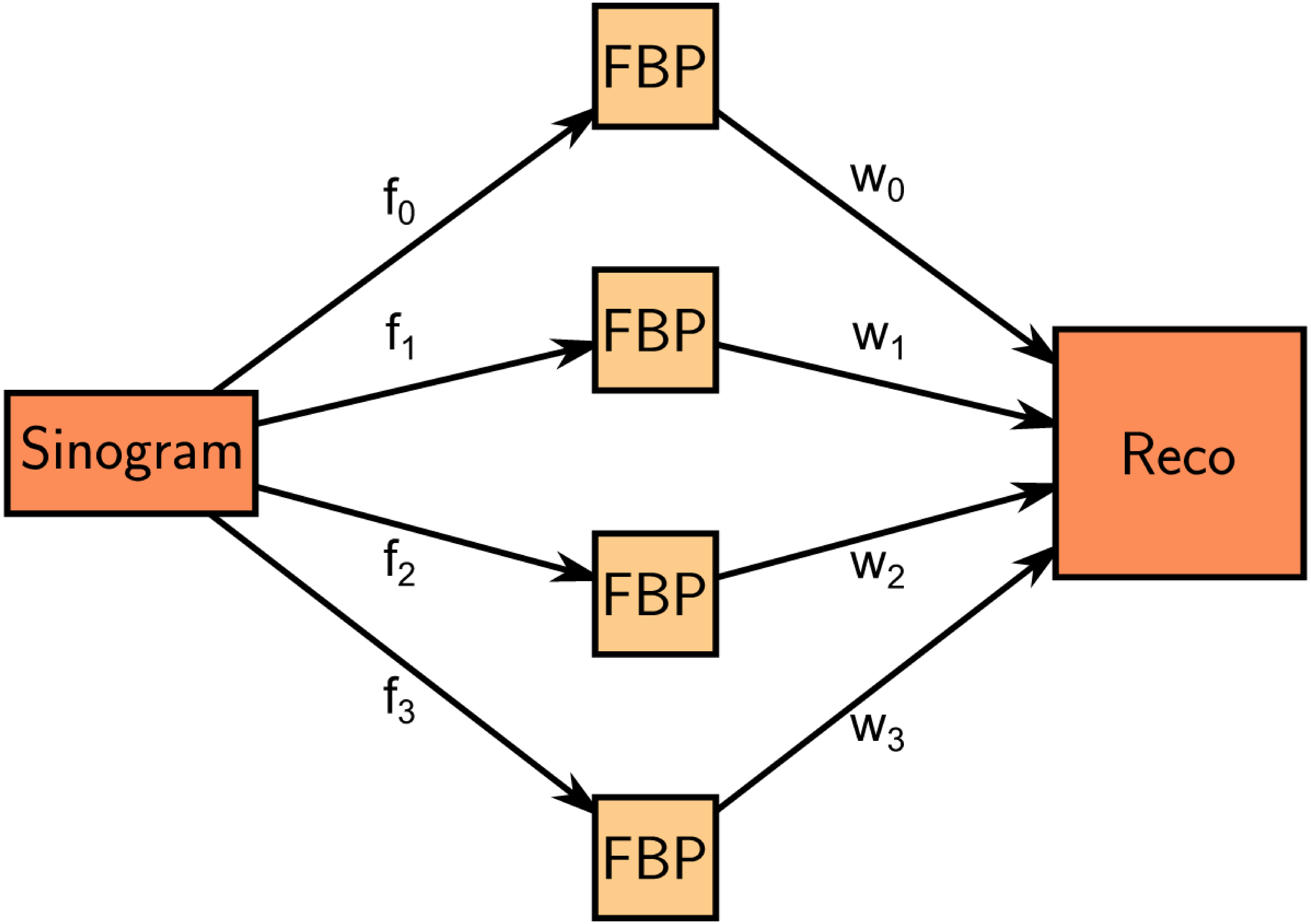
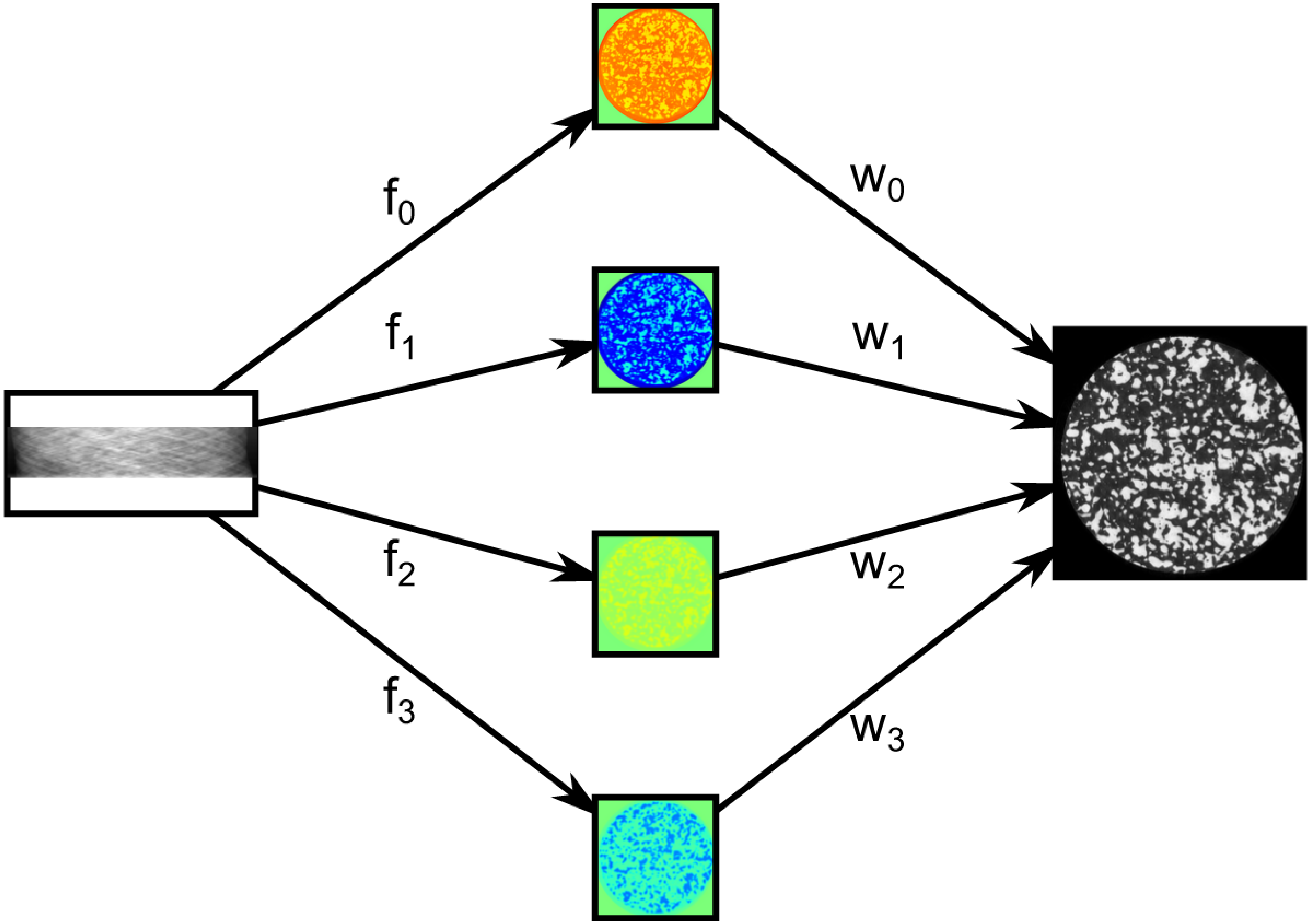
Approach #3: Neural networks



Approach #3: Neural networks

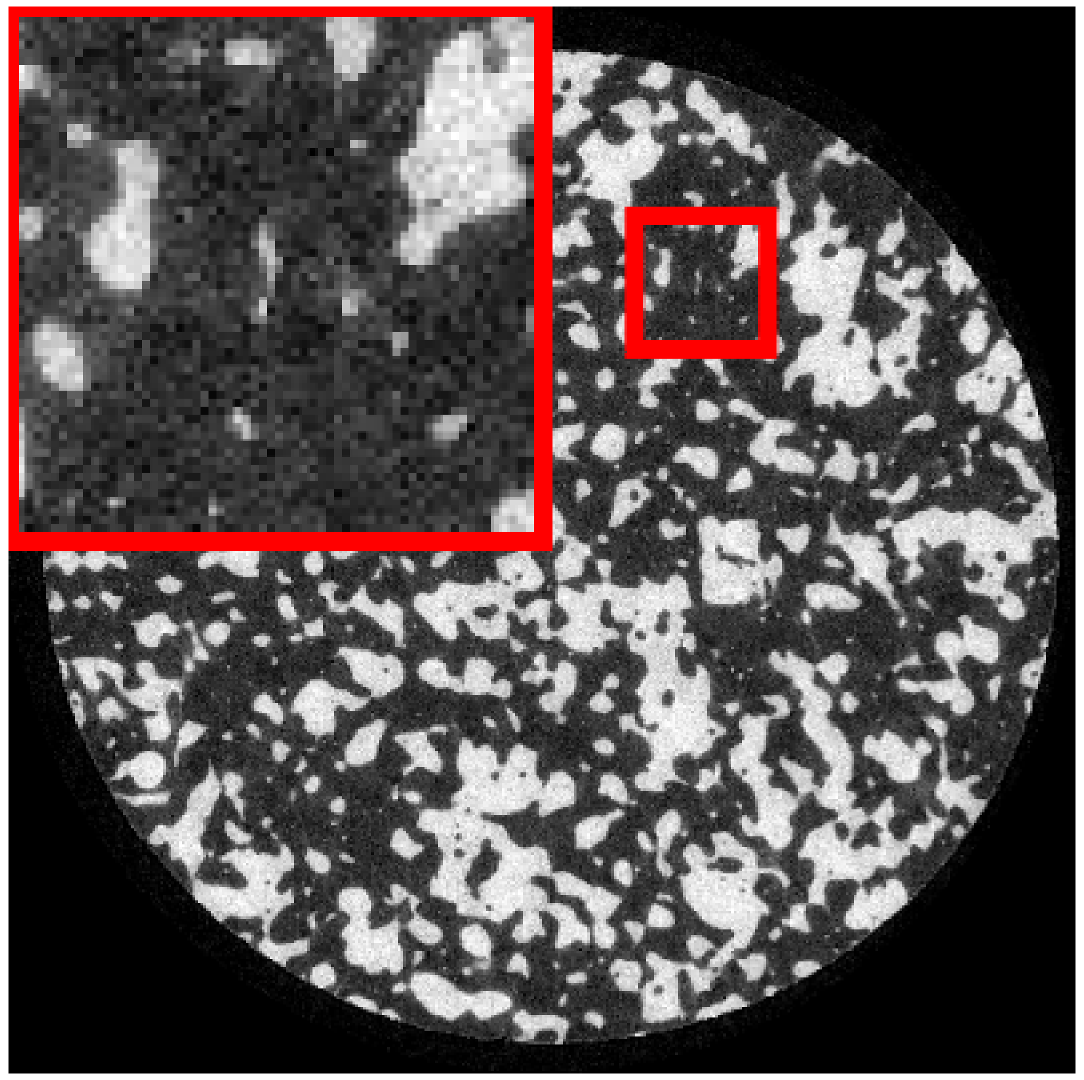
- Filters and weights are trained using neural network theory
- To train, high-quality reconstructions of objects are needed
  - Scan representative objects with high dose
  - Scan at the start/end of a dynamic experiment
  - ...
- The network will learn filters that exploit:
  - Acquisition details (noise profile, # projections, ...)
  - Object characteristics
- After training, reconstruction is fast and accurate

Approach #3: Neural networks

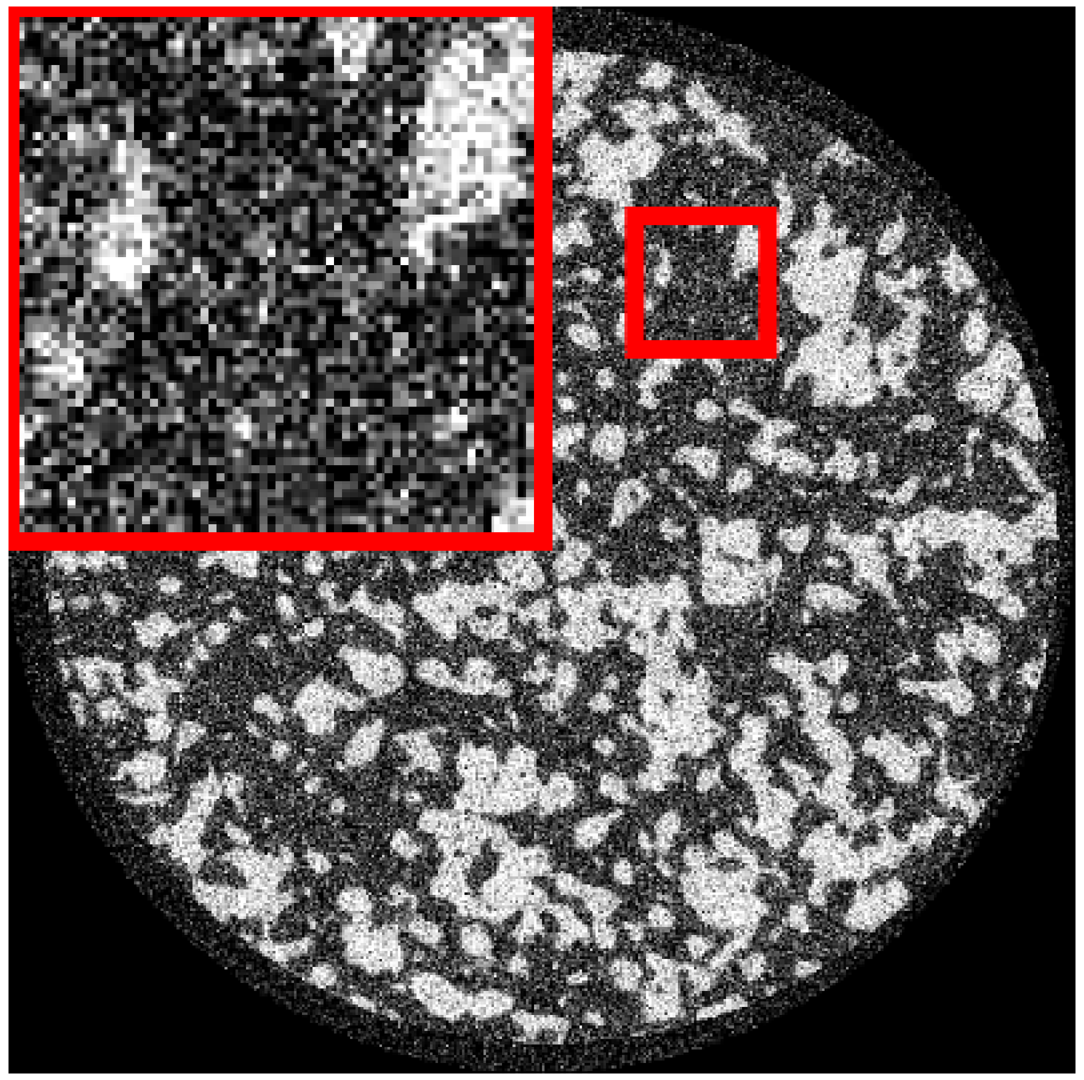


Approach #3: Neural networks

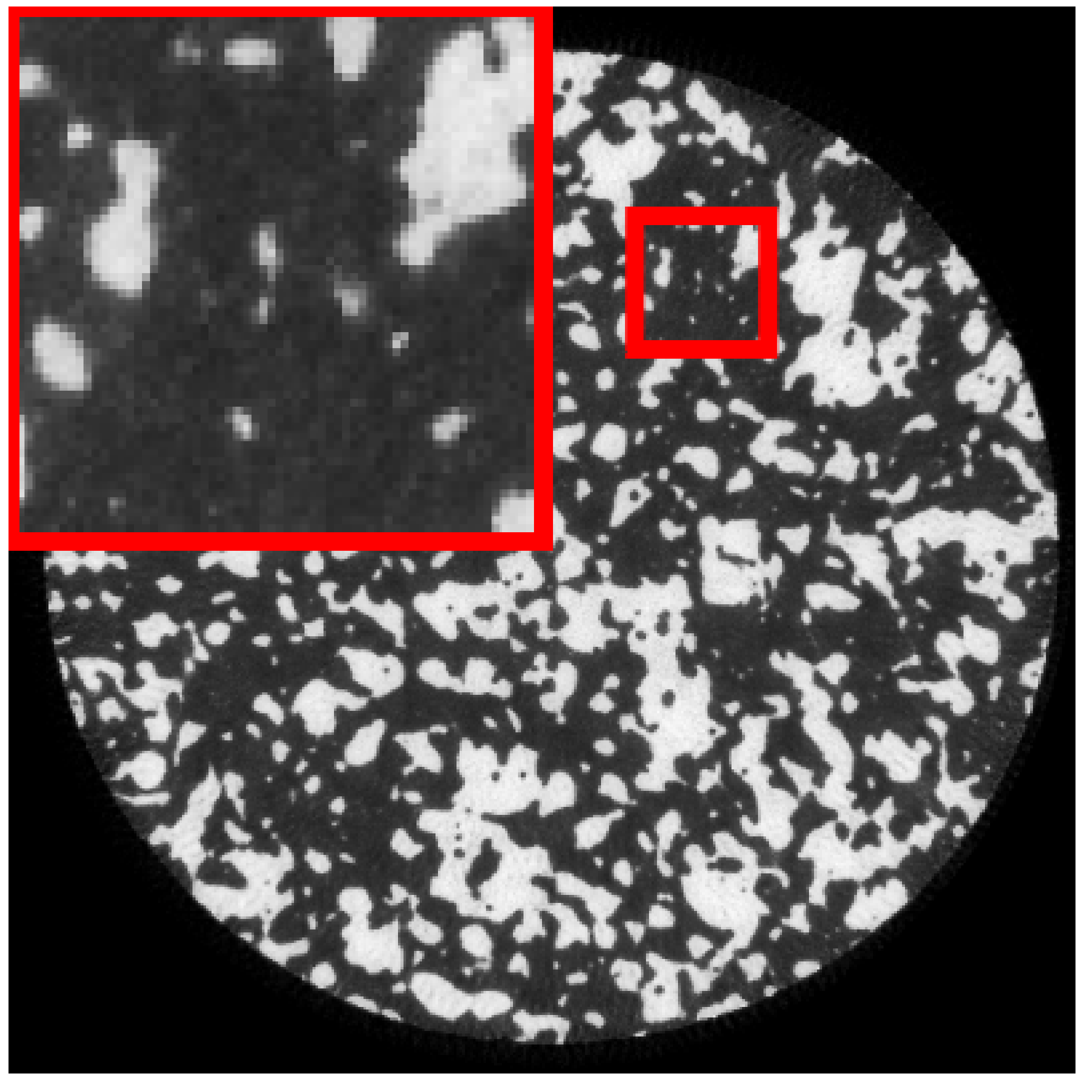
4k x 4k pixels, synchrotron data (ESRF)



(a) FBP (all projections)

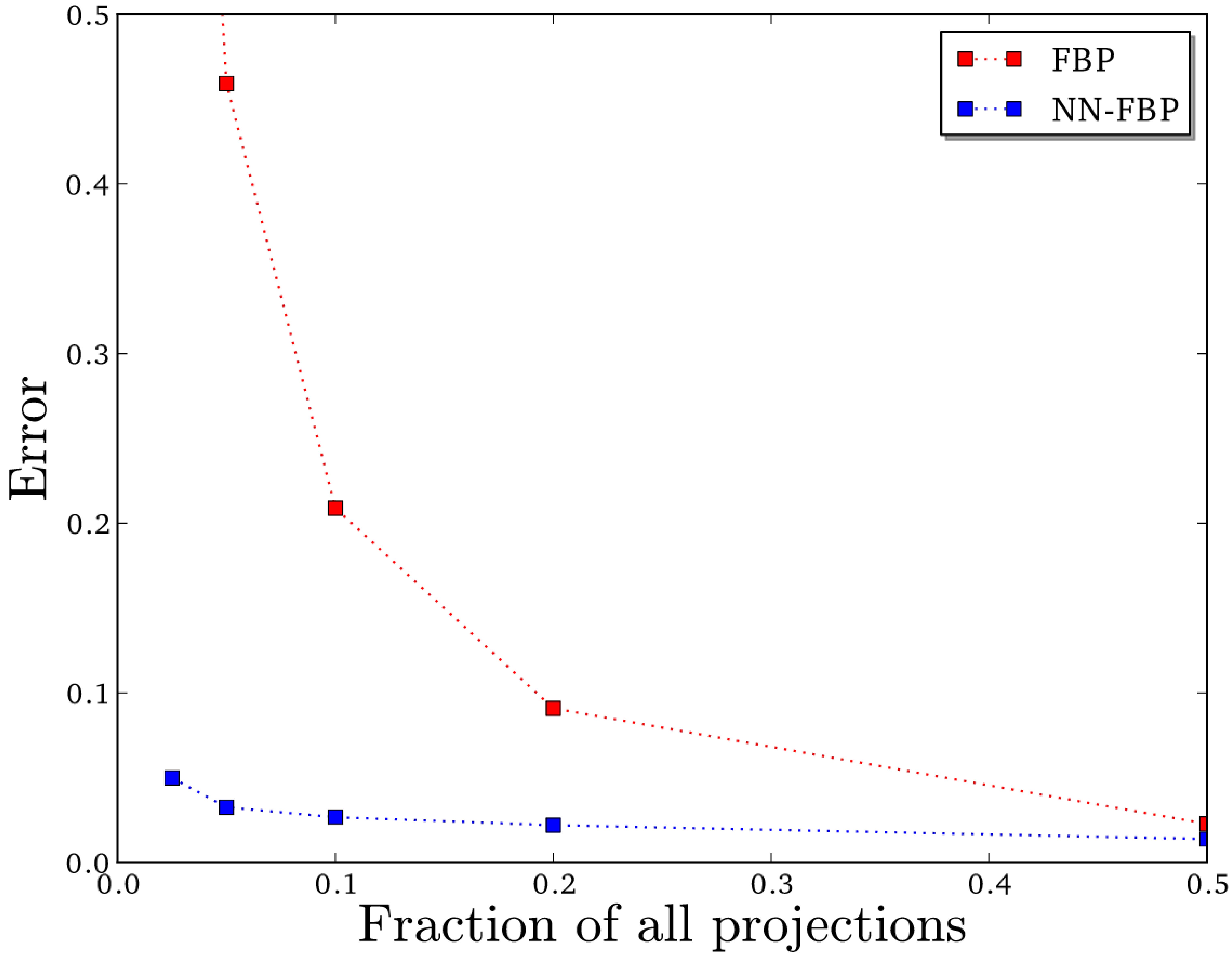


(b) FBP (5%)



(c) NN-FBP (5%)

Approach #3: Neural networks



Conclusions

- FBP with non-standard filters can produce very accurate reconstructions
- The filter can be chosen in different ways, each with advantages and disadvantages
- MR-FBP
  - Use a data-dependent filter that minimizes the projection error
- SIRT-FBP
  - Use a filter that approximates an algebraic method
- NN-FBP
  - Train filters using high-quality training datasets

Thank you for listening!

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Open source implementations available at: https://github.com/dmpelt/

References:

[1] Pelt, D. M., & Batenburg, K. J. (2014). Improving Filtered Backprojection Reconstruction by Data-Dependent Filtering. *Image Processing, IEEE Transactions on, 23*(11), 4750-4762.

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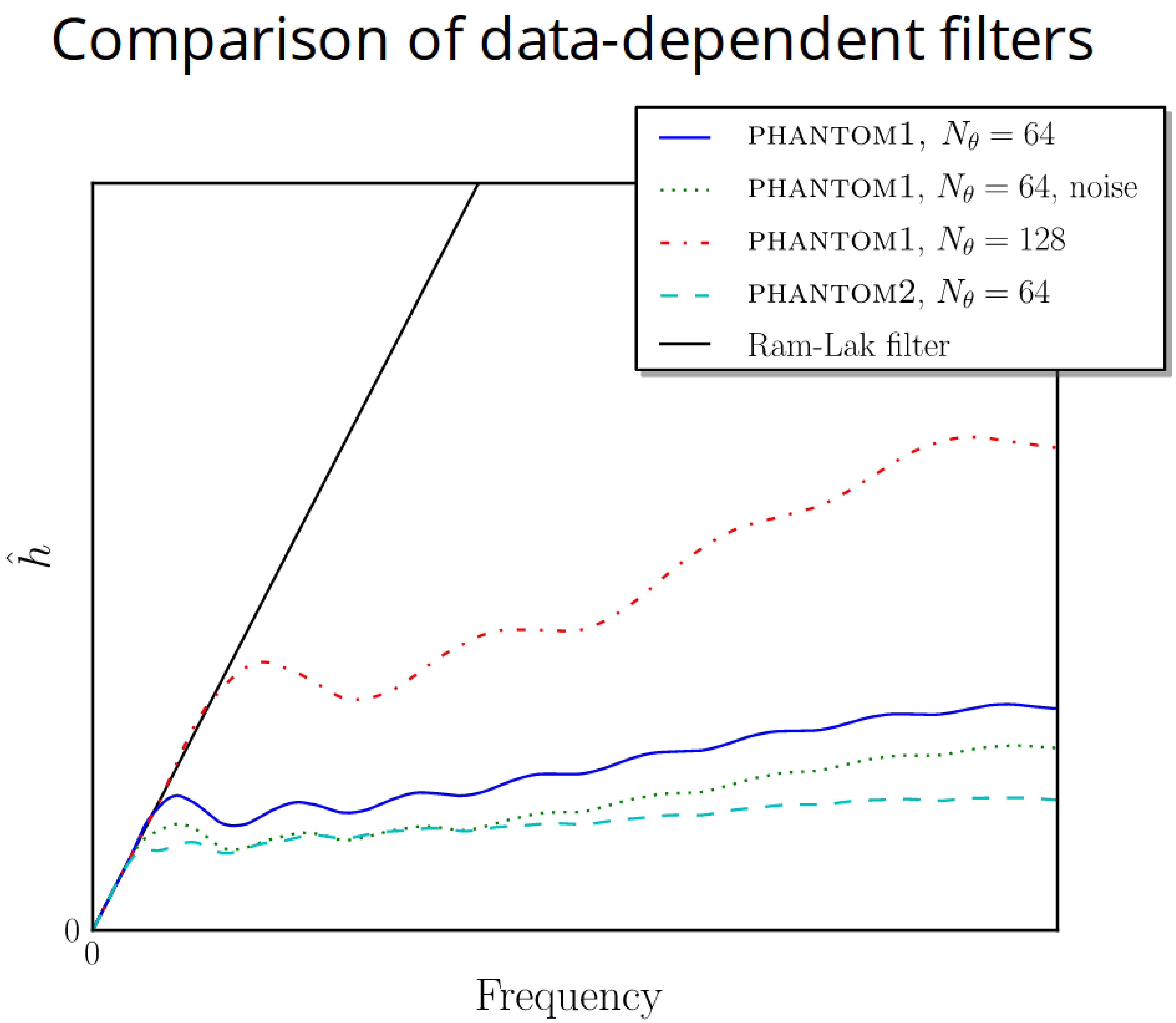

Approach #3: Neural networks

Trained filters