Tomographic reconstruction: the challenge of “dark” information

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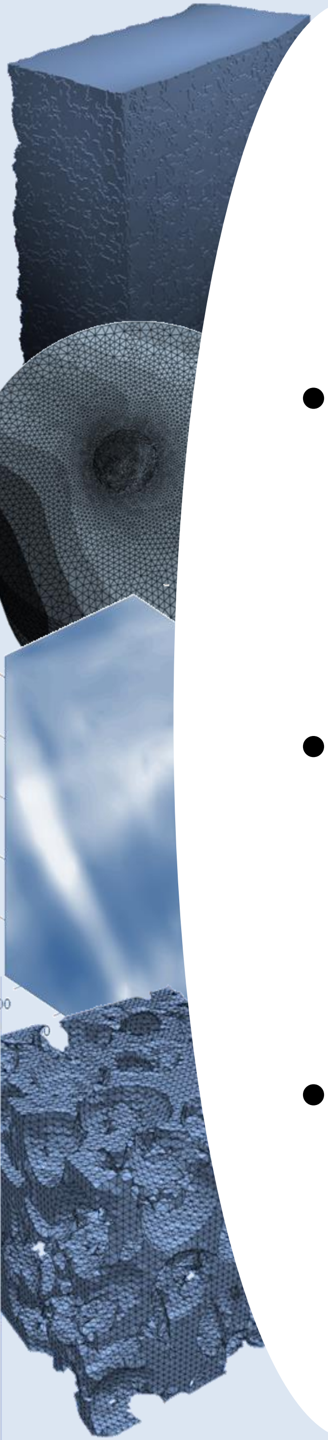
Tomography

- A mature technique, providing an outstanding wealth of information from medical applications to material science

Tomography

- Yet, outstanding progress has been accomplished over the past few years
  - Ultra-High spatial resolution
  - Ultra-Fast
  - Finer sensitivity (Phase contrast)
  - Enriched tomography (DCT)



Tomography

- In parallel, further developments involve a tremendous amount of data
- Motivation is high to reduce the quantity of needed information
- Can we do more with less ?

Big data

- A key element in the context of images is the information content
- A 1 Mpix. gray scale image encoded over 8 bits requires

$$2^{10} \times 2^{10} \times 2^{3} = 2^{23} = 8{,}388{,}608$$

bits of information

Big data

- A key element in the context of images is the information content
- A 1 Mpix. gray scale image encoded over 8 bits requires

$$2^{10} \times 2^{10} \times 2^{3} = 2^{23} = 8{,}388{,}608$$

bits of information

- **Really ?**

Compression

- Some bits have more “value” than others!
- A certain amount of compression can be harmless (or even beneficial)

Celebrated example

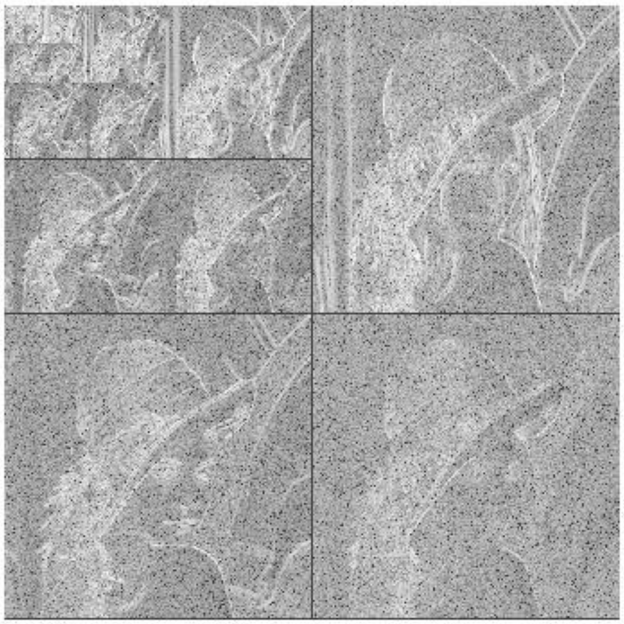
- “Lenna” original

Celebrated example

- “Lenna” 70% compression (Daubechies wavelets)

Data Compression

- Compression efficiency depends on the chosen *representation* of the image and scale



Data Compression

- Compressibility relies on a possibly *sparse* representation of an image:
  for a suited basis, only few elements are non-zero

- Compressibility also means a large potential for noise reduction

Efficient measurement

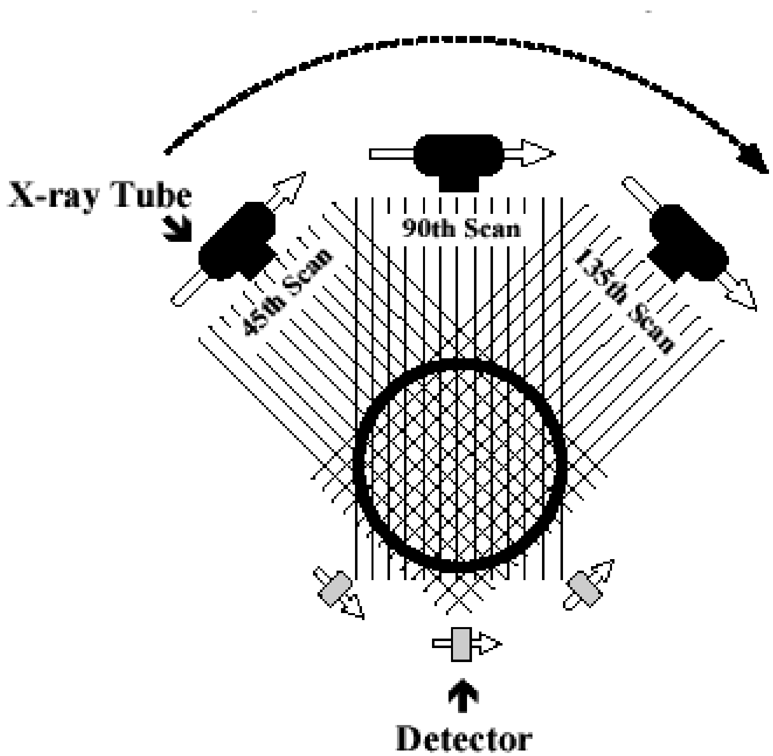
- Can we do something smarter and only "read" the information needed to capture the sparse components rather than acquiring all and throwing away the least meaningful?

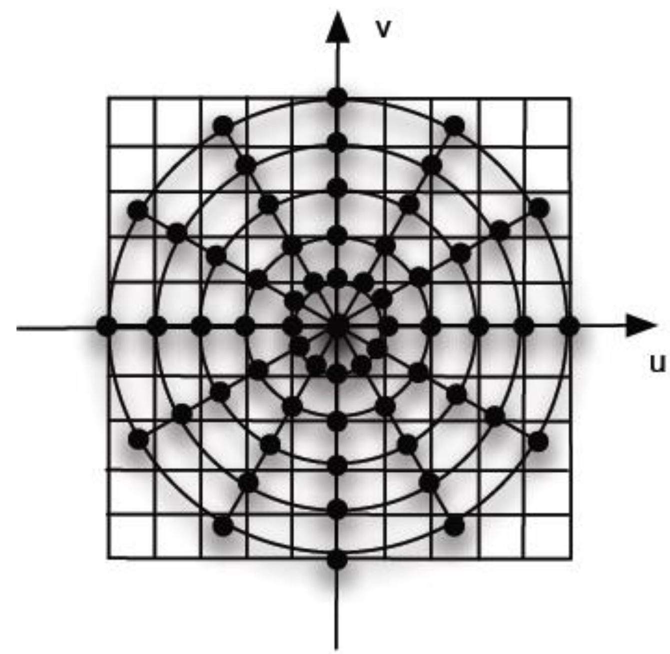
- This is the field of *Compressed sensing*

The case of tomography

- Candés, Romberg and Tao (2004)

Reconstruction

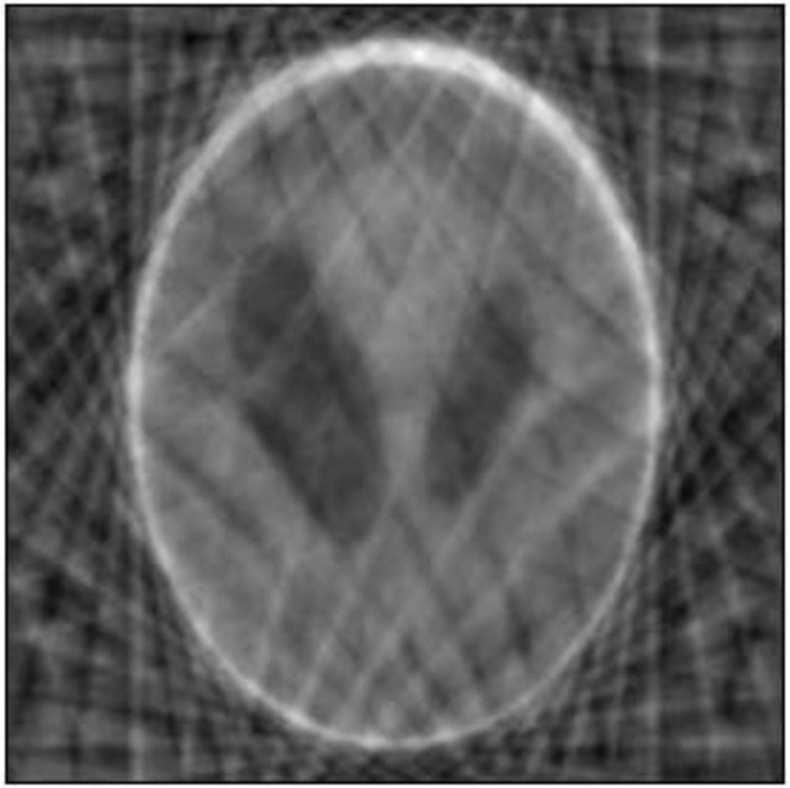




In Fourier space

The case of tomography

- Classical reconstruction from 22 projections (rather than 600)



The case of tomography

- Suited reconstruction based on 22 projections minimizing total variation

Complexity

- In this example, $\left|\nabla f\right|$ is *sparse*
- $f(\boldsymbol{x})$ can be reconstructed as the solution to

$$f(\boldsymbol{x}) = \underset{\Pi_\theta g = \pi_\theta}{\operatorname{argmin}} \int \left|\nabla g(\boldsymbol{x})\right| \mathrm{d}\boldsymbol{x}$$

  convex minimization problem (provided *enough* projections are considered)
- The trick is to use a L1 norm

Opportunities

- Exploit *prior information* on material under study (number of phases, geometry, …)
- Resort to sparse representation as an intermediate step in iterative algebraic reconstruction

Opportunities

- Sparsity is typically not shared by noise. Hence finding a sparse representation of a signal is naturally a *denoising* technique

- Reconstruction is well suited to multi-scale approaches

What are the limits ?

Classically

- 600 projections are well-suited for FBP reconstruction for a 400×400 images
- # of inputs is 400×600 data
- # of outputs is 400×400 data

Slightly redundant (150%)

What are the limits ?

TV minimization

- 22 projections are sufficient for a 400×400 images
- # of inputs is 400×22 data
- # of outputs is 400×400 data

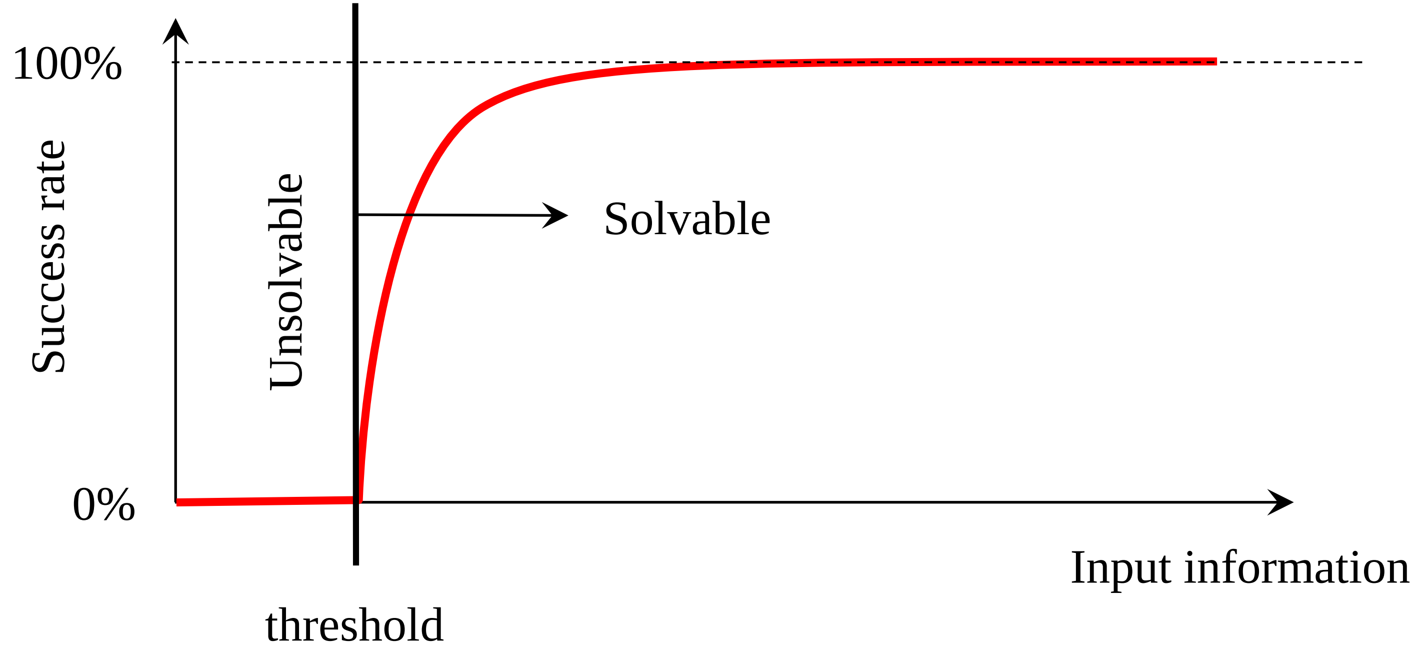
Significantly underdetermined (5.5%) without additional "dark" information

What are the limits ?

- Why not go to lower values ?

A phase transition

- Donoho & Tanner's phase transition
  - (sparsity = few non-zero pixels)



A phase transition

Number of pixels N^2

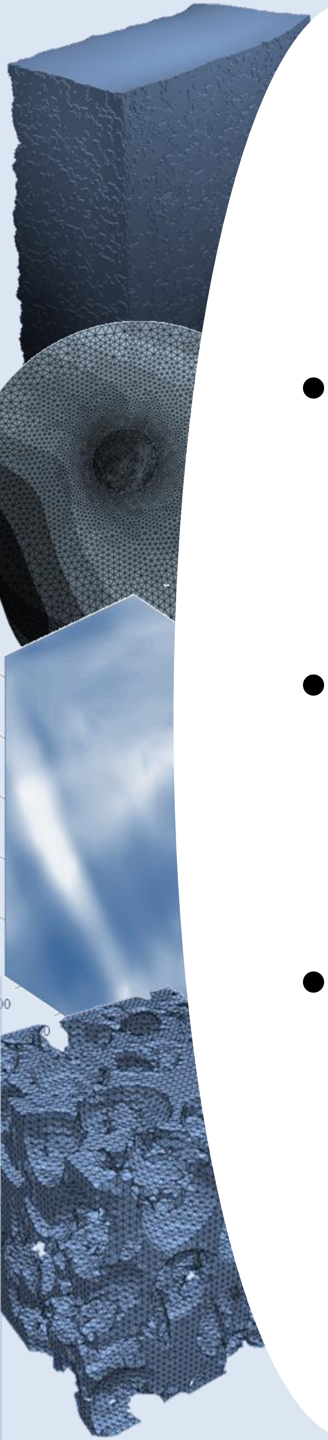
Number of projections M

Fraction of non-zero pixels p

Solvability condition $\quad p < p_c$

$$p_c \approx \frac{M}{2N \log(N/M)}$$

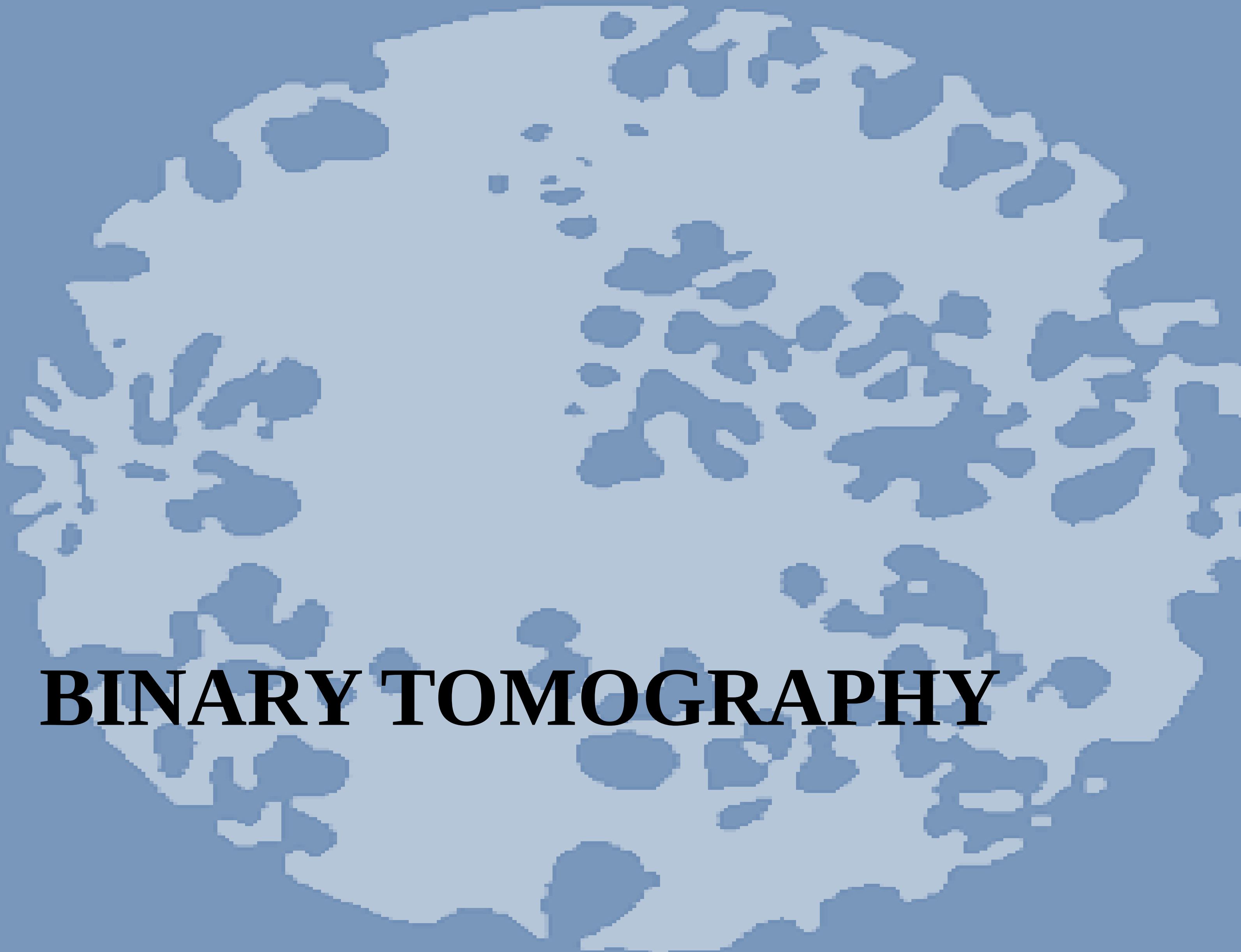
Donoho & Tanner, Disc. Comp. Geom. (2010)



Open questions

- What is the least number of parameters to describe accurately the tomographied specimen ?
- Can one combine a suited sparse representation with efficient reconstruction techniques ?
- Is there an objective measure of complexity that would dictate the number of projections to be used ?

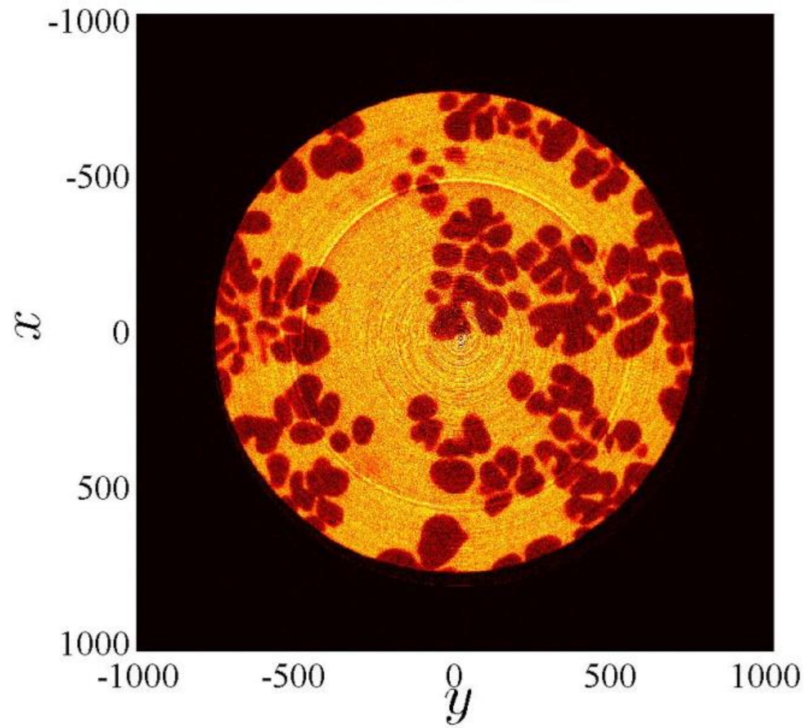
BINARY TOMOGRAPHY



SR, Leclerc, Hild, J. Math. Im. Vis. (2013)

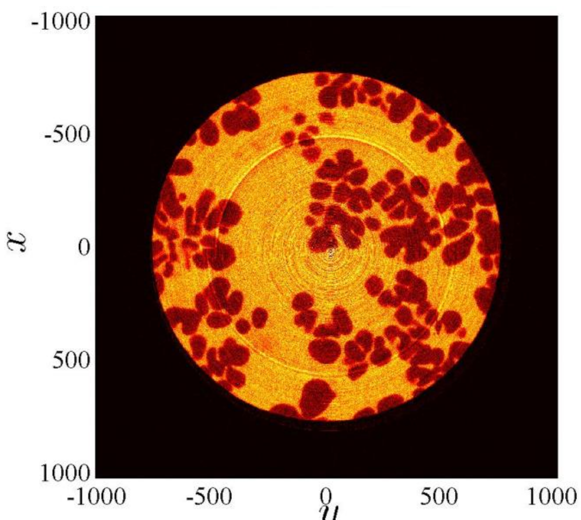
An example from fast tomography

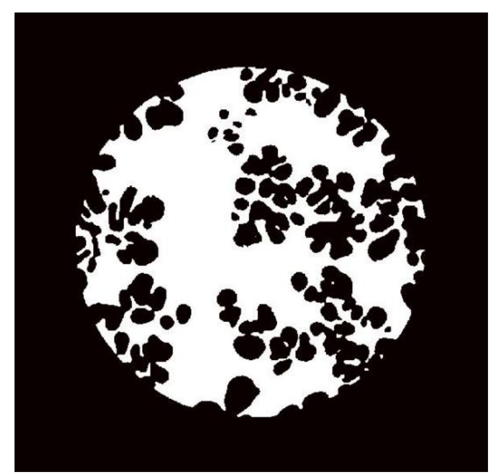
- Al-Cu phase separation images at 0.14 s (1500 projections) European Synchrotron Radiation Facility (E. Gouillart, L. Salvo)



Binary reconstruction

- A priori information on the reconstructed field may reduce drastically the needed information (projection)
- An example of such prior information is the reconstruction of binary fields





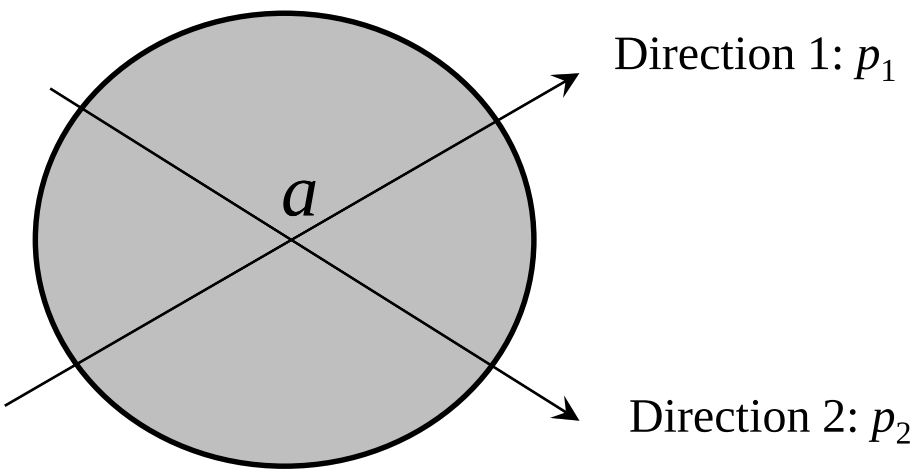
Proposed algorithm

- A: Initialize with a first guess
- B: Iteratively enhance the matching with projections

- Very efficient algorithm leading to *error-free* reconstructions for large image sizes (1000×1000), few projections (4-20), in seconds (10-20 iterations)

A: Initialization

- Probability that a particular site is valued 1: p
- Known from projection i: p_i $\qquad q_i = 1 - p_i$



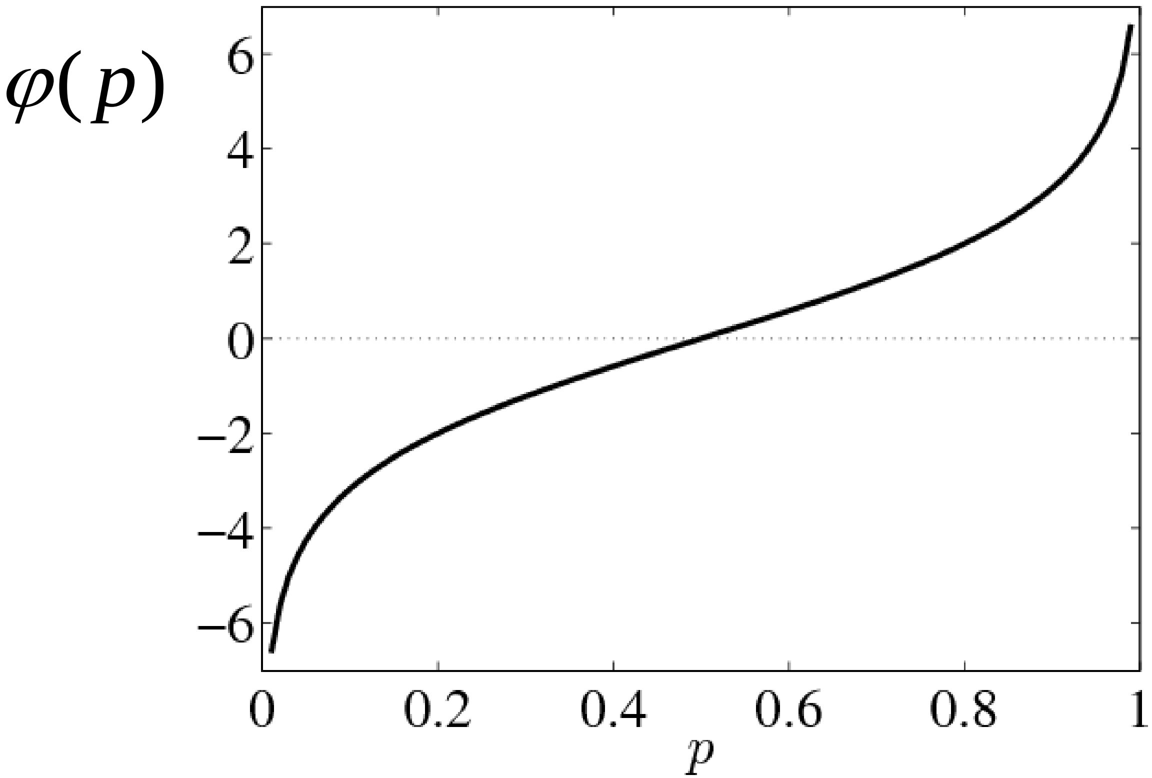
$$p = \frac{p_1 p_2}{p_1 p_2 + q_1 q_2}$$

A: Initialization

- A nice property:
  - Introduce $$\varphi(p) = \log\left(\frac{p}{1-p}\right) \qquad p = \frac{e^{\varphi}}{1+e^{\varphi}}$$
  - Then $$\varphi(p) = \varphi(p_1) + \varphi(p_2)$$
  - Holds for an arbitrary number of projections: FBP can be used on φ

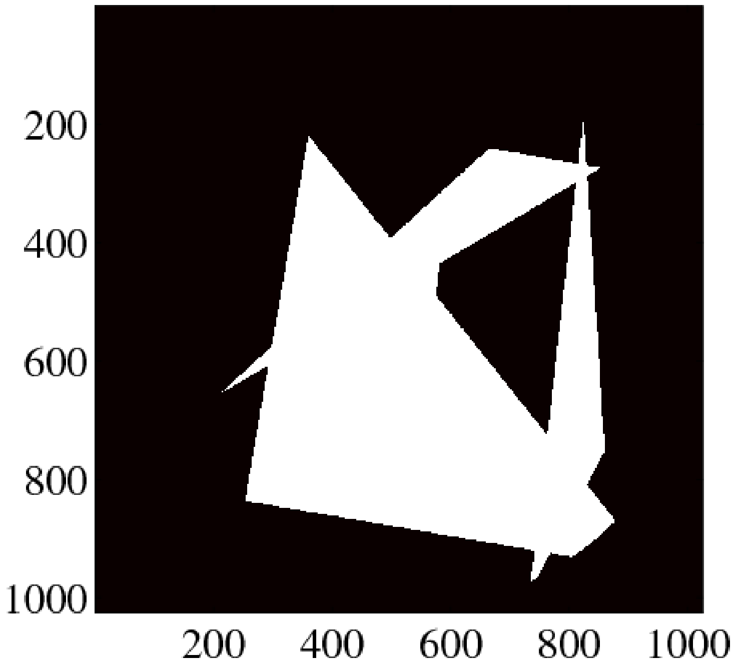
Non-linear transform

- Transform

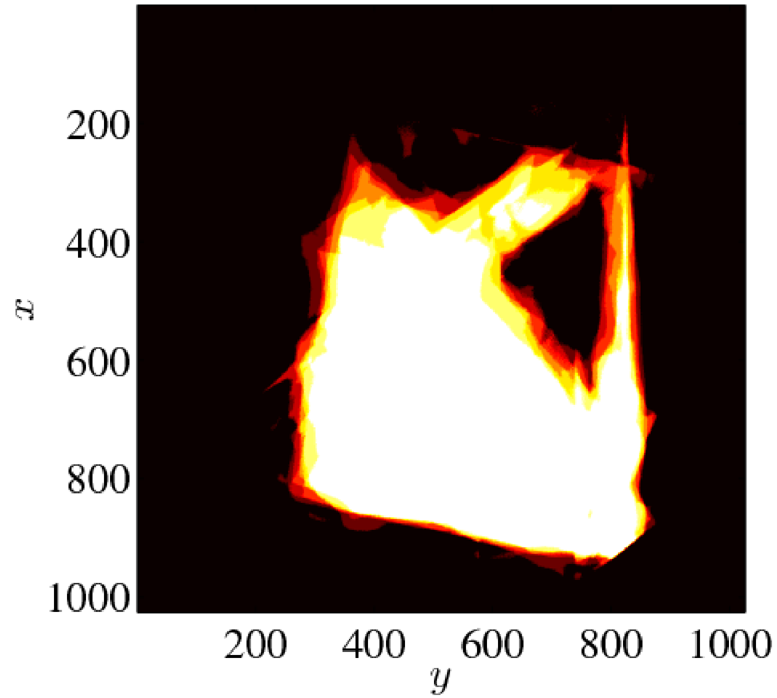


An example

Original image

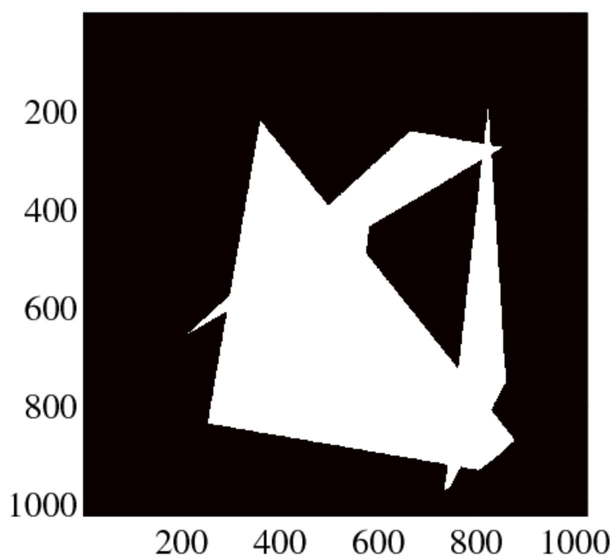


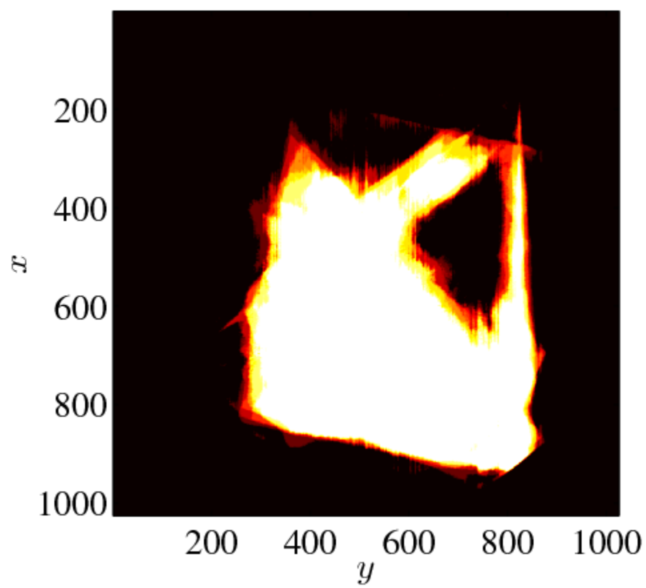
φ-FBP with 7 projections



Initialization step

- Lowest (<0.01) and highest (>.99) probabilities can be set to 0 and 1 respectively, and a second pass of the same FBP can be used on the undetermined sites





ART algorithm

- Projection constraint for each projection direction j $W^{j} \cdot f = \pi^{j}$

- ART correction step

$$f^{(n+1)} = f^{(n)} + B^{j} \cdot (\pi^{j} - W^{j} \cdot f^{(n)})$$

(uniform additive correction along each ray)

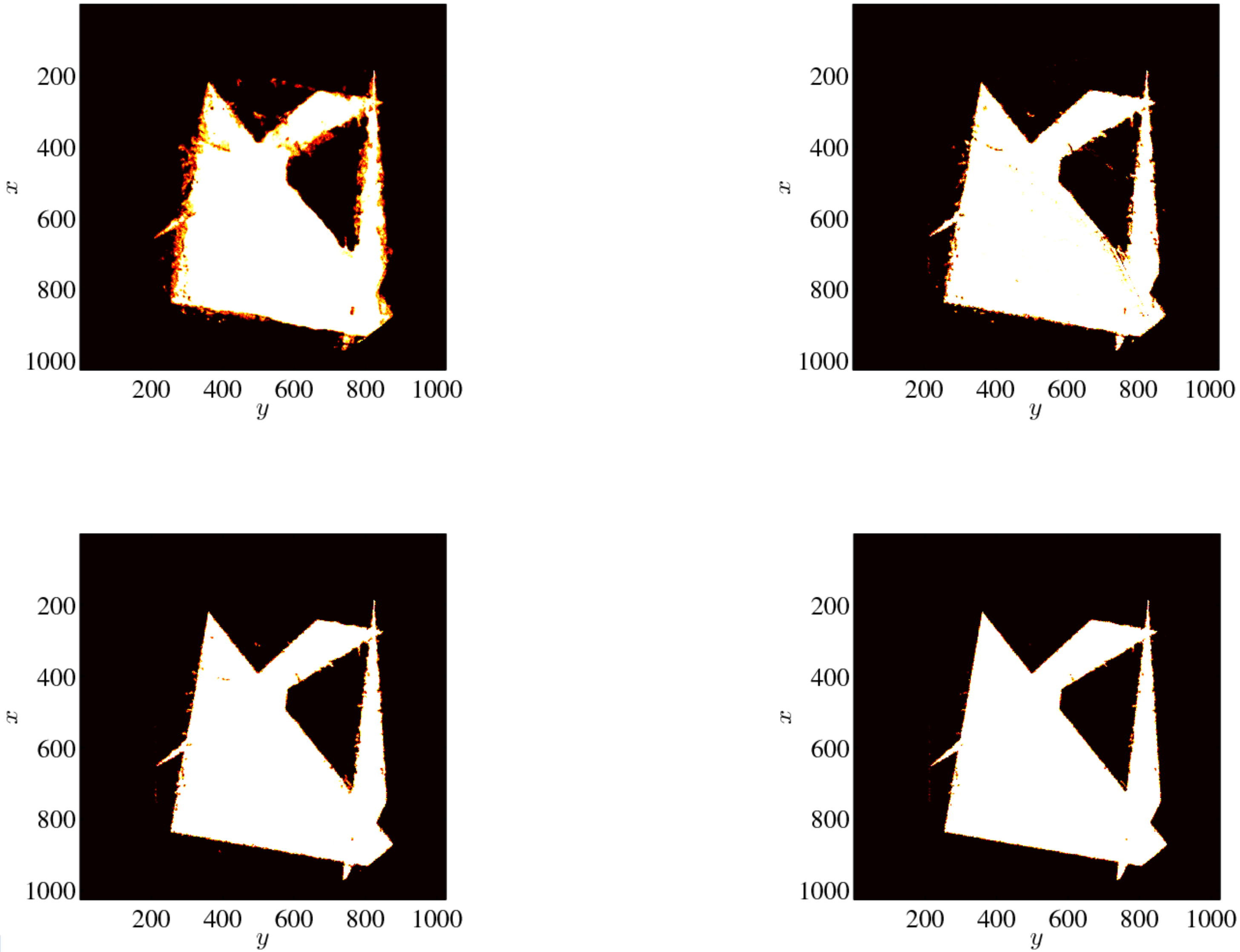
Other variants

- MART is a Multiplicative variant (*f* is scaled along rays to meet the projection)
- Note that this variant is an ART correction on log(*f*)
- Thus a natural idea is to design a similar strategy on $\varphi(f)$

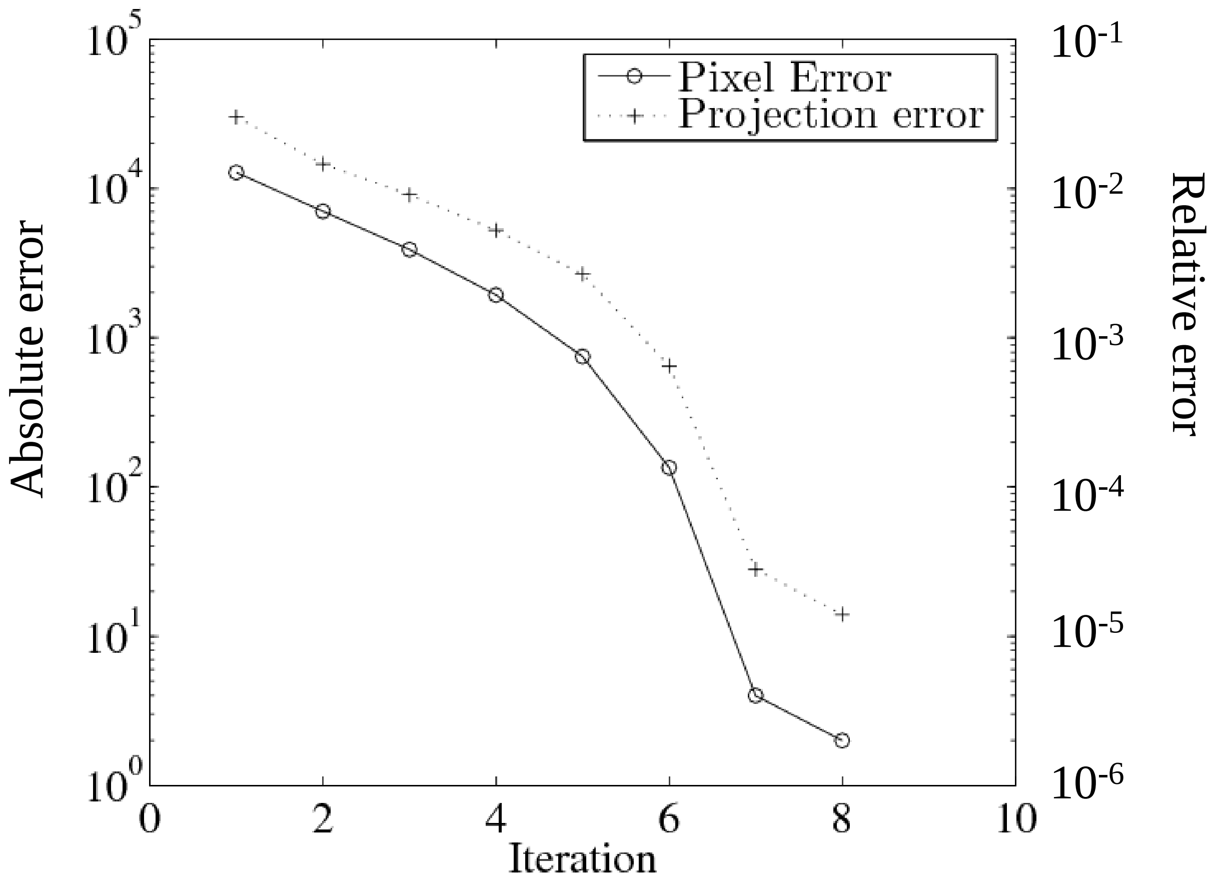
Correction step

- For each direction *j*, and individual rays, the $\varphi(f)$ values are translated so that a binarization would lead to the exact projection: This amounts to sorting out the π^j largest values of $\varphi(f)$, and translate all $\varphi(f)$ so that the π^jth value is set to 0
- After having visited all directions, *f* is binarized and convoluted with a Gaussian

Correction step



Convergence rate



Conclusion on the example

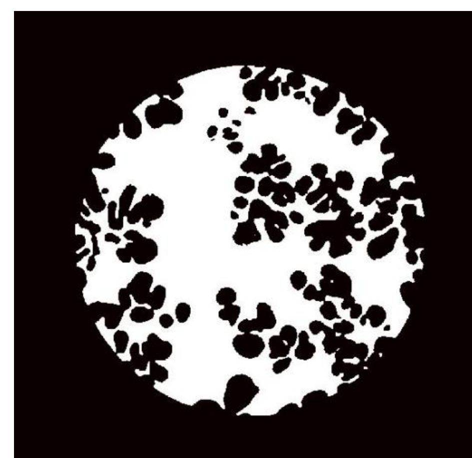
- More than 10^6 unknowns
- 7 projections: 7×10^3 data (0.7 %)
- Exact solution reached in 4 iterations
- Smoothness parameter: *r* geometrically decreasing with iteration number from 3 to 1
- Computation time (Matlab®, single processor, no optimization): 8.4 s

Multiscale

- The same algorithm can be used on a coarse-grained image where 2 × 2 pixels are grouped together (majority rule). The resulting image is used as a seed for the correction steps at the finer scale
- This scaling feature can be used recursively

A complex microstructure case

- Single scale: 31 iteration steps, 189s, for an error free reconstruction on 1 Mb image.

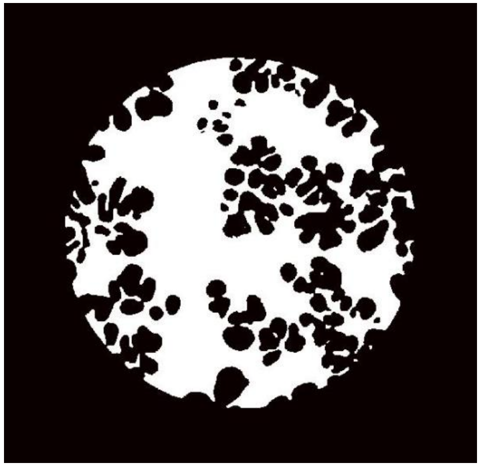


A complex microstructure case

- Single scale: 31 iteration steps, 189s, for an error free recosntruction on 1 Mb image.
- Multiscale: 5 levels, 28 s, or 7 times faster

Generation = 4	Error = 0	Iteration = 3
Generation = 3	Error = 0	Iteration = 8
Generation = 2	Error = 0	Iteration = 6
Generation = 1	Error = 0	Iteration = 3
Generation = 0	Error = 0	Iteration = 3

Total time = 28 s

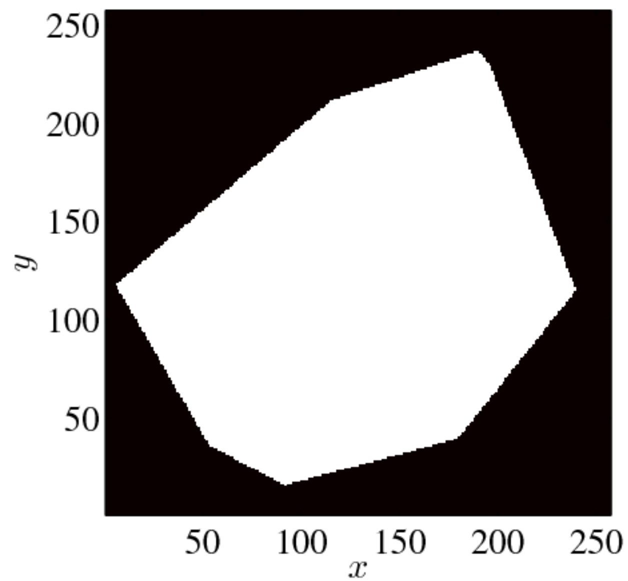


Other examples

Statistics over 200 random samples

- Single convex polygon

Number Proj.	Perfect reconst.	Pixel error	Time (s)
3	93%	3	0.36
4	99%	0.6	0.45

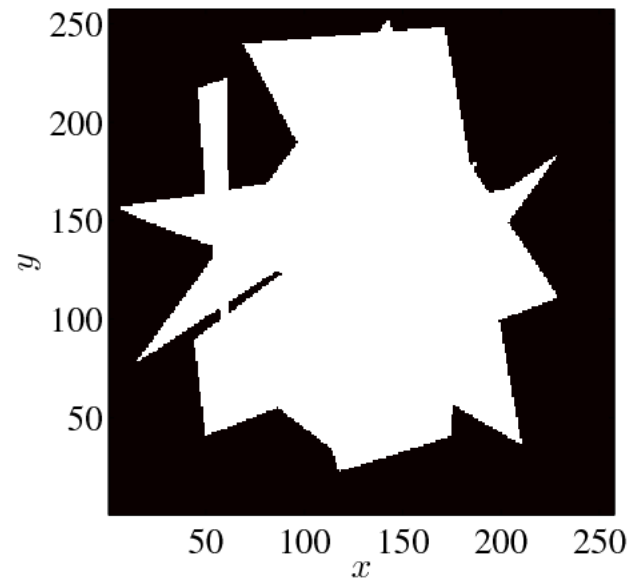


Other examples

Statistics over 200 random samples

- Several convex polygons

Number Proj.	Perfect reconst.	Pixel error	Time (s)
4	90%	21.0	1.92
5	97%	1.3	1.31
6	100%	0	1.04

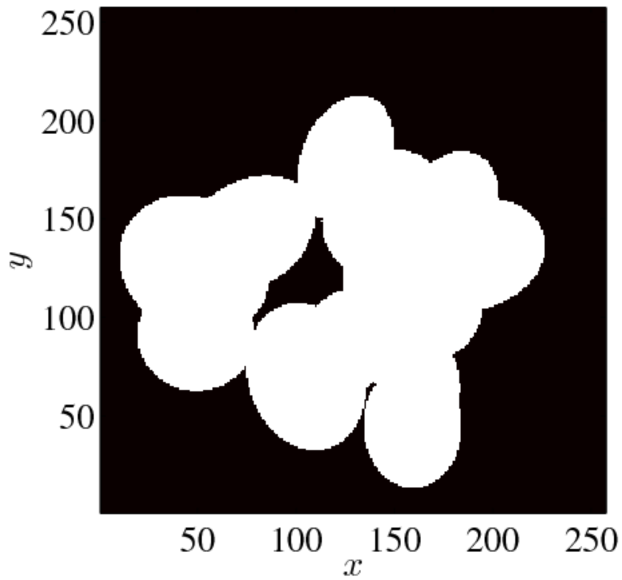


Other examples

Statistics over 200 random samples

- Several ellipses

Number Proj.	Perfect reconst.	Pixel error	Time (s)
4	83%	41	2.0
5	99%	0.005	1.4
6	100%	0.	1.3

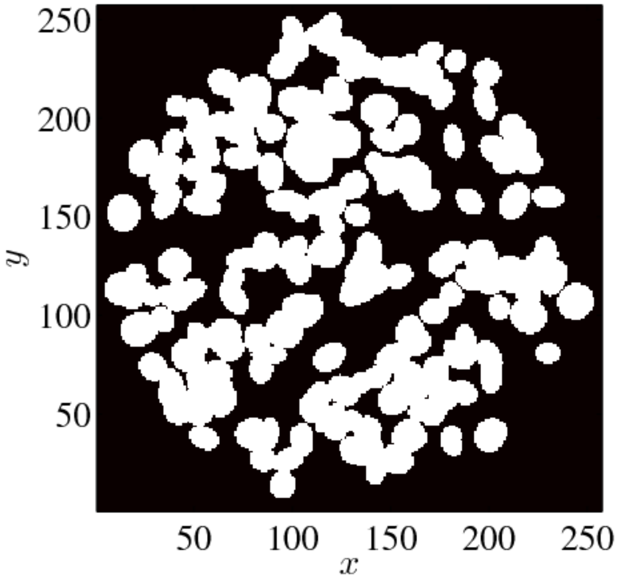


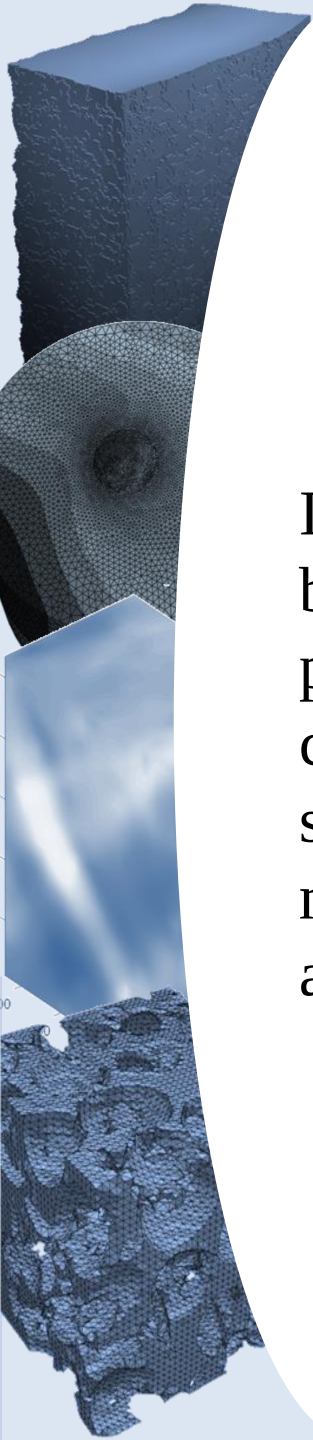
Other examples

Statistics over 200 random samples

- Many small ellipses

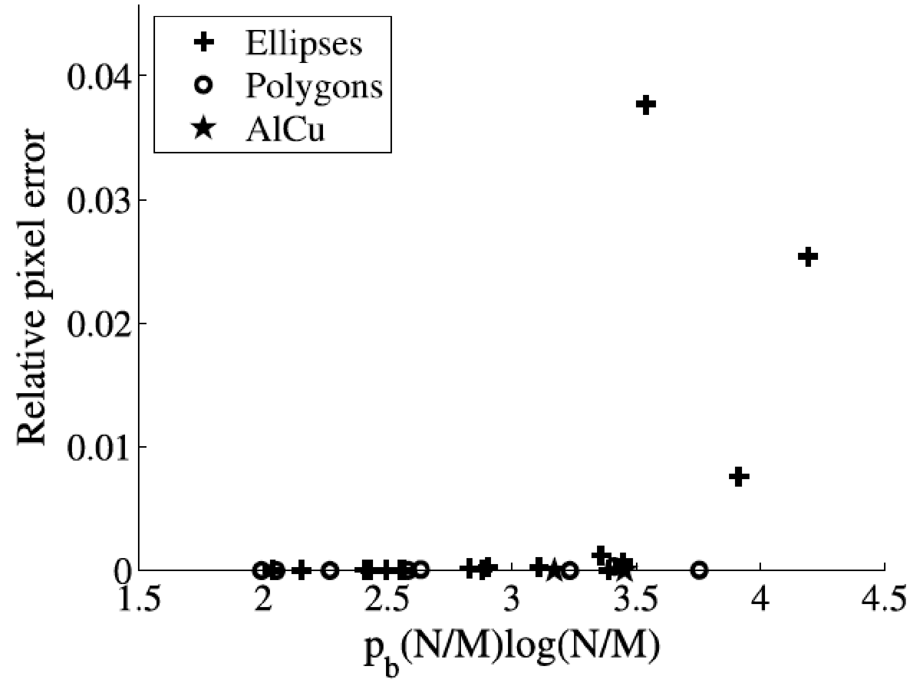
Number Proj.	Perfect reconst.	Pixel error	Time (s)
14	98%	5.	14.1
16	98%	5.	14.8





Number of projections ?

If the fraction p_b of boundary pixels is a proper measure of complexity, a solvability criterion may be guessed to amount to



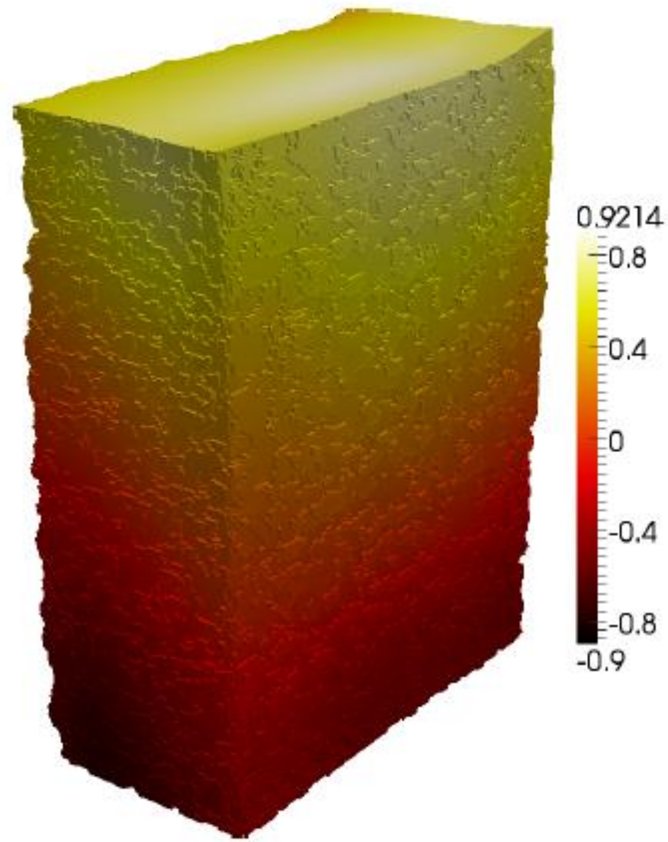
$$\frac{N \log(N / M)\, p_b^{crit}}{M} \approx cst$$

DIGITAL VOLUME CORRELATION WITHOUT RECONSTRUCTION

How to measure a 3D displacement field efficiently?

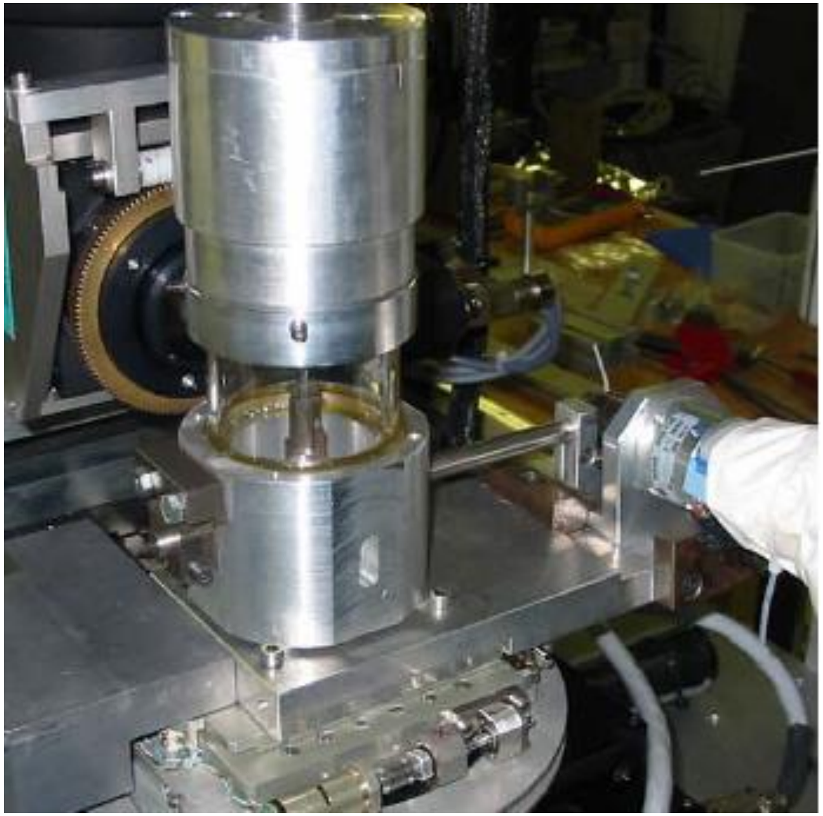
- Test case:
- Nodular cast iron specimen

- ID19 @ ESRF
- Energy = 60 keV



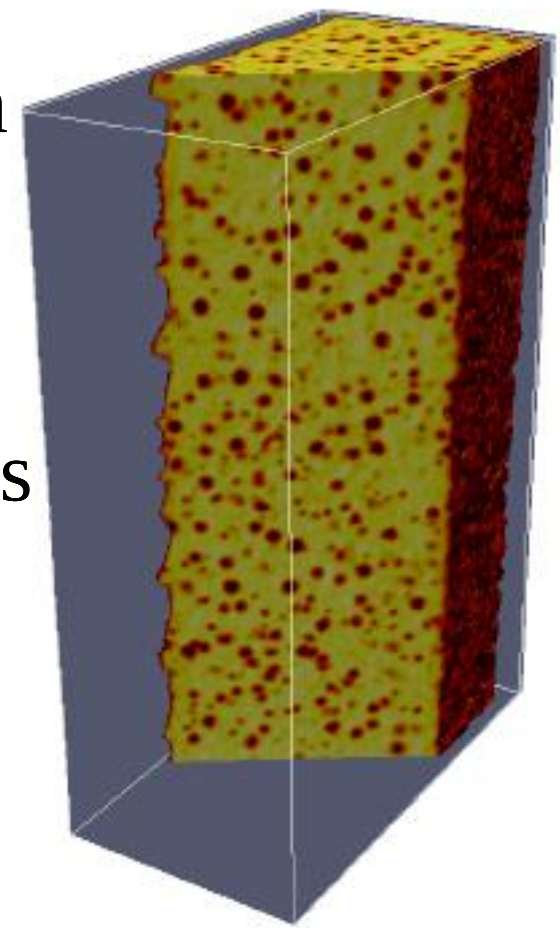
in-situ mechanical testing

- In-situ tensile test (E. Maire & J. Y. Buffière's testing machine)
- Acquisition of two 3D images (elastic regime)



Material

- Nodule size 50 µm
- Inter nodule distance 50µm
- Rough surface
- Voxel = 5.1 µm
- ROI = 180×330×400 voxels



Reconstruction

- Reconstruction

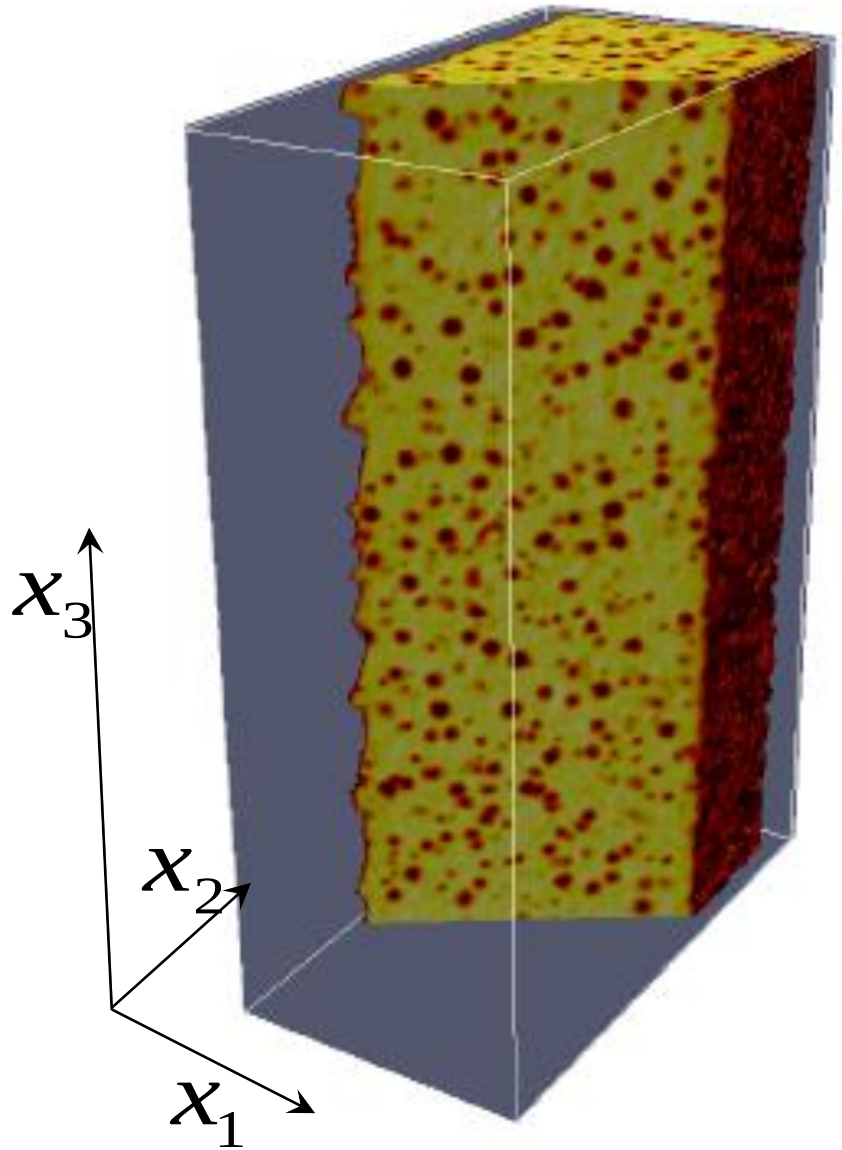
$$f(\boldsymbol{x}) = \mathcal{R}\left[s(\boldsymbol{r},\phi)\right]$$

$$f = \mathcal{R}[s]$$

- Projection

$$s(\boldsymbol{r},\phi) = \mathcal{P}_{\phi}\left[f(\boldsymbol{x})\right]$$

$$s = \mathcal{P}_{\{\phi\}}\left[f\right]$$



DVC

- Starting from two images $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$, of the same object at two different loading conditions, DVC consists of measuring the displacement field $\boldsymbol{u}(\boldsymbol{x})$ from

$$f_2(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x})) = f_1(\boldsymbol{x})$$

Global-DVC

- The displacement field is decomposed over a library of displacement fields

$$\boldsymbol{U}(\boldsymbol{X}) = \sum_i a_i \boldsymbol{\Psi}_i(\boldsymbol{X})$$

- As an example, finite-element shape functions can be chosen

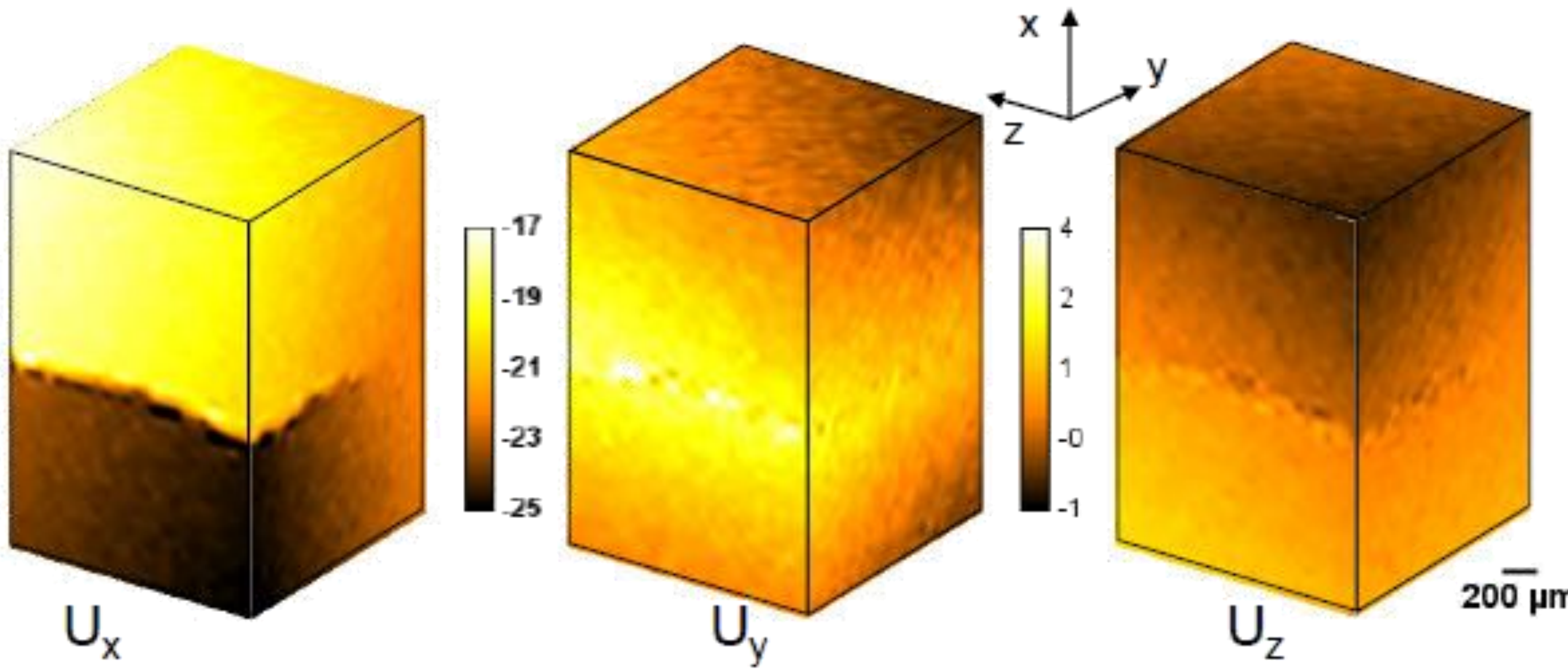
Global DVC

- The amplitudes, a_i , are determined from the minimization of

$$a = Argmin_b \iiint \left(f_2(\boldsymbol{X}) - f_1(\boldsymbol{X} - b_i \boldsymbol{\Psi}_i(\boldsymbol{X})) \right)^2 \mathrm{d}\boldsymbol{X}$$

- Multiscale and/or mechanical regularization strategy to manage robustness and basin of convergence.

DVC Displacement field



- N G Cast Iron Fatigue Crack
- C8 DVC

1 voxel ↔ 3.5 µm

*[*N.Limodin et al, Acta Mat. 57, 4090, (2009)*]

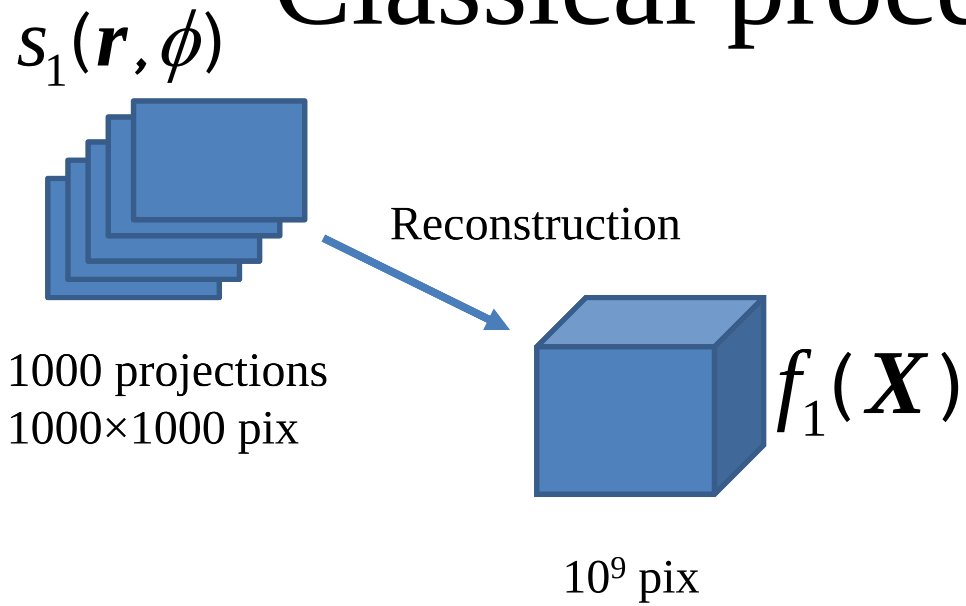
Global DVC vs reconstruction

- For a 10×10×10 vox. cubic mesh,

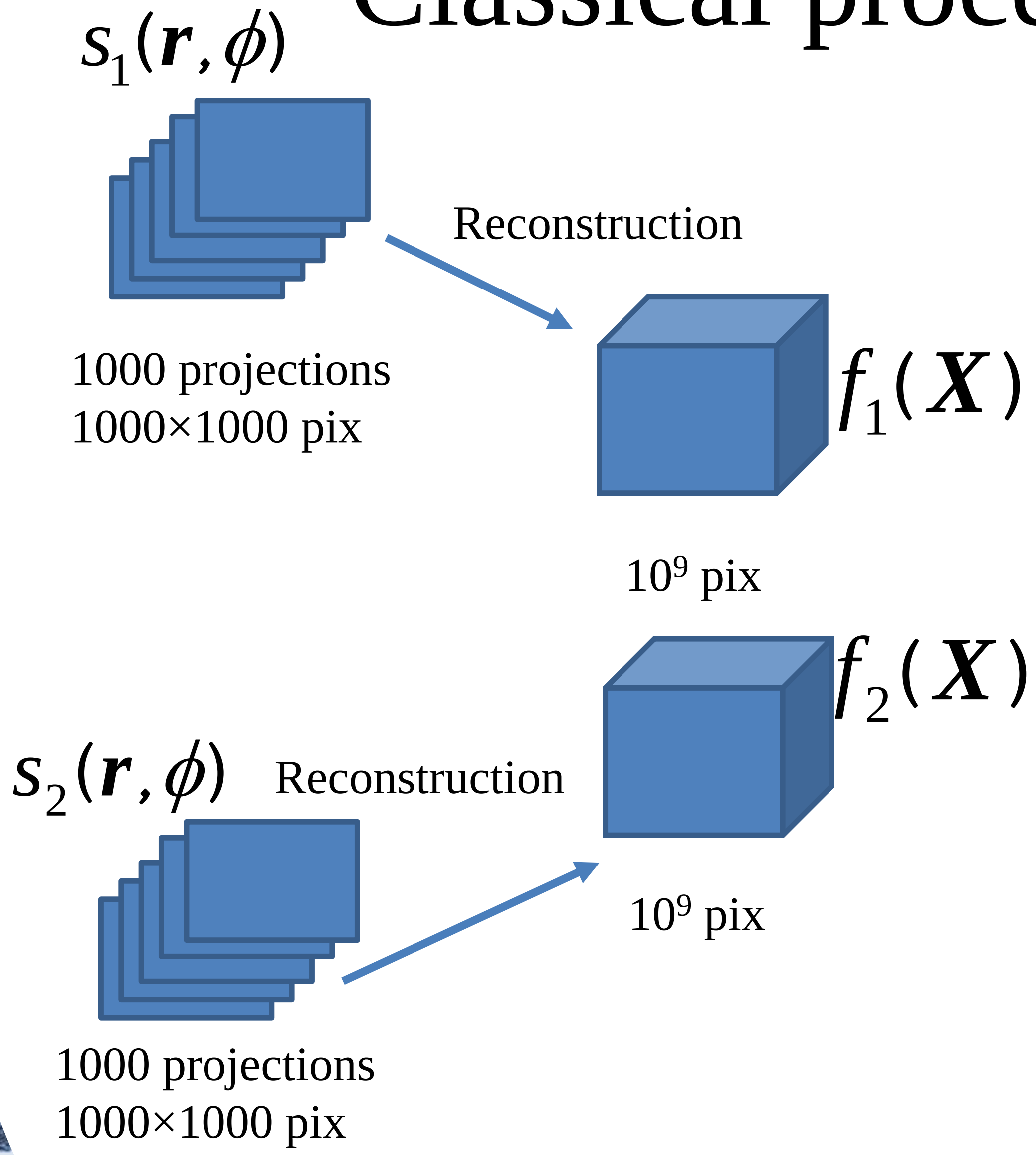
$$\frac{\text{Number of kinematic unknowns}}{\text{Number of voxels}} = 0.003 \ll 1$$

- A factor of more than 100 could be saved on the number of projections !

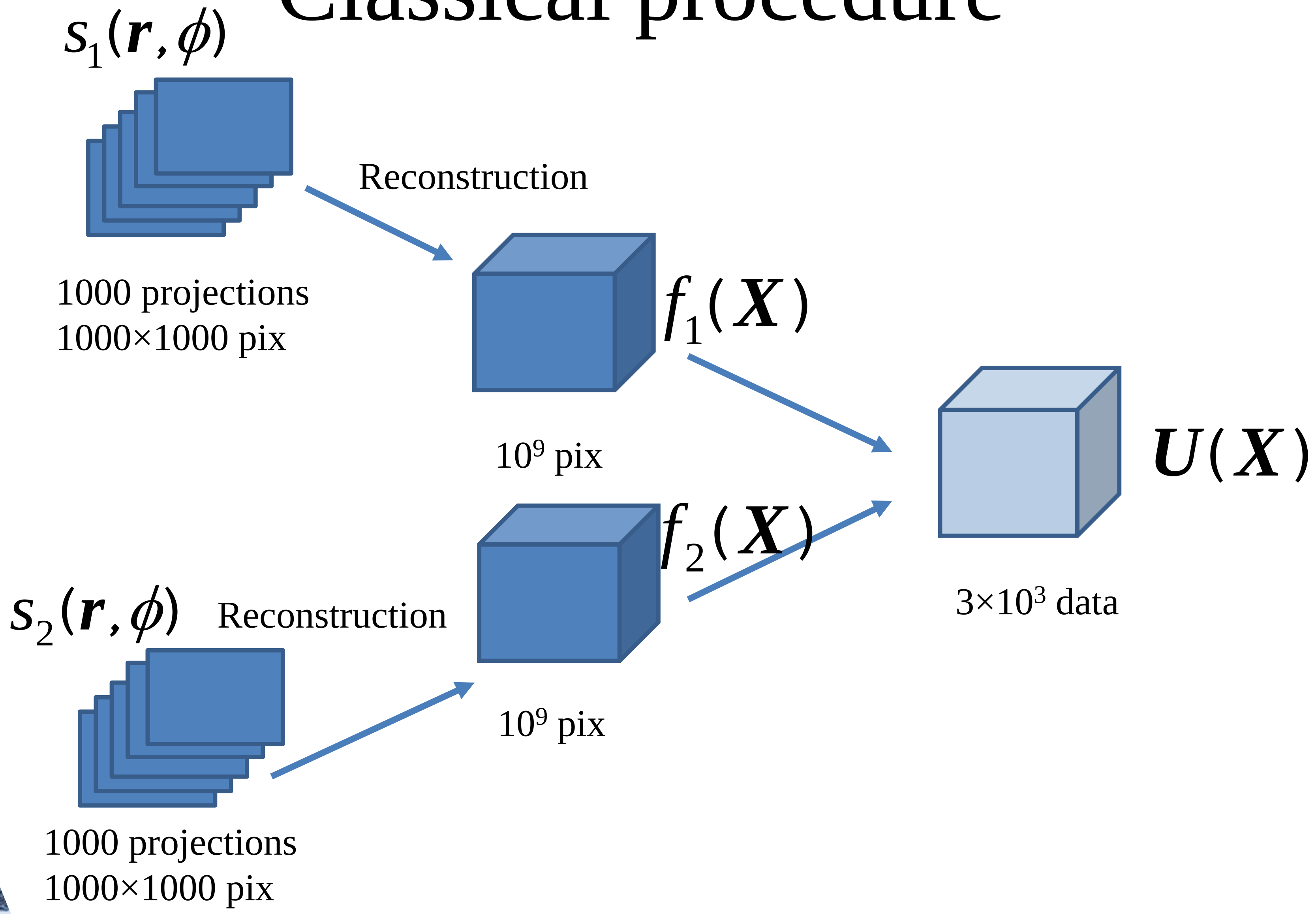
Classical procedure



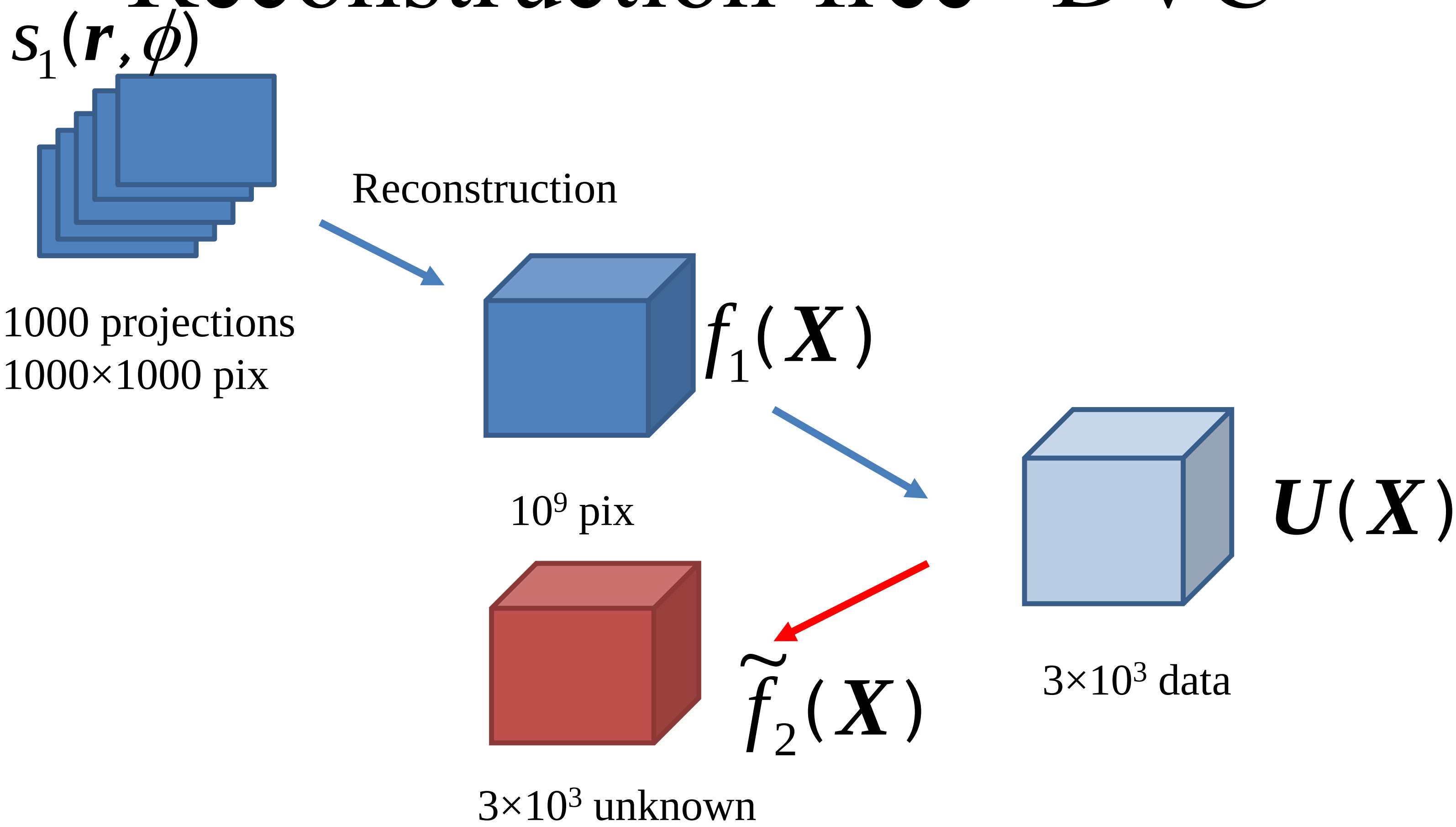
Classical procedure



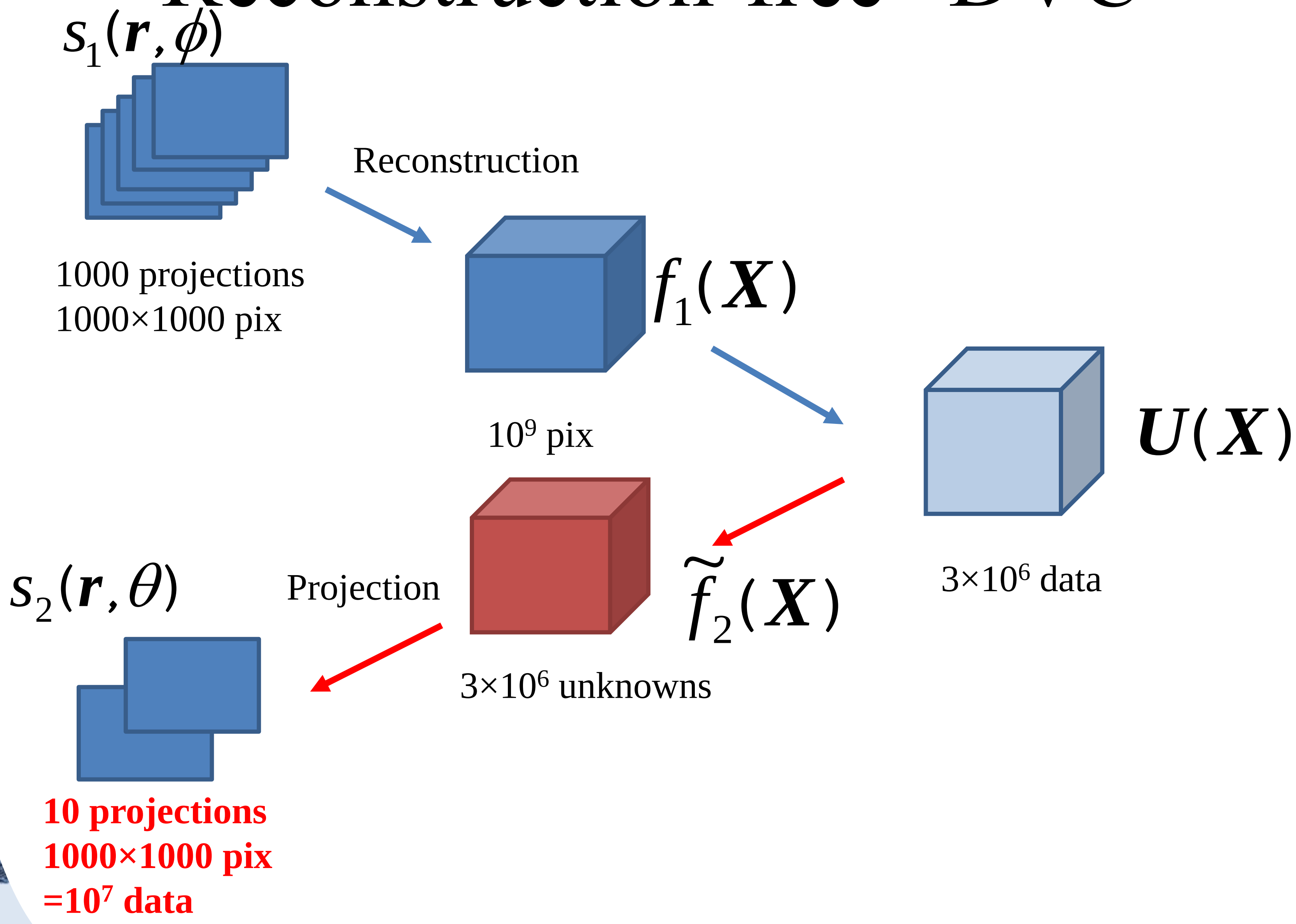
Classical procedure



“Reconstruction-free” DVC



“Reconstruction-free” DVC



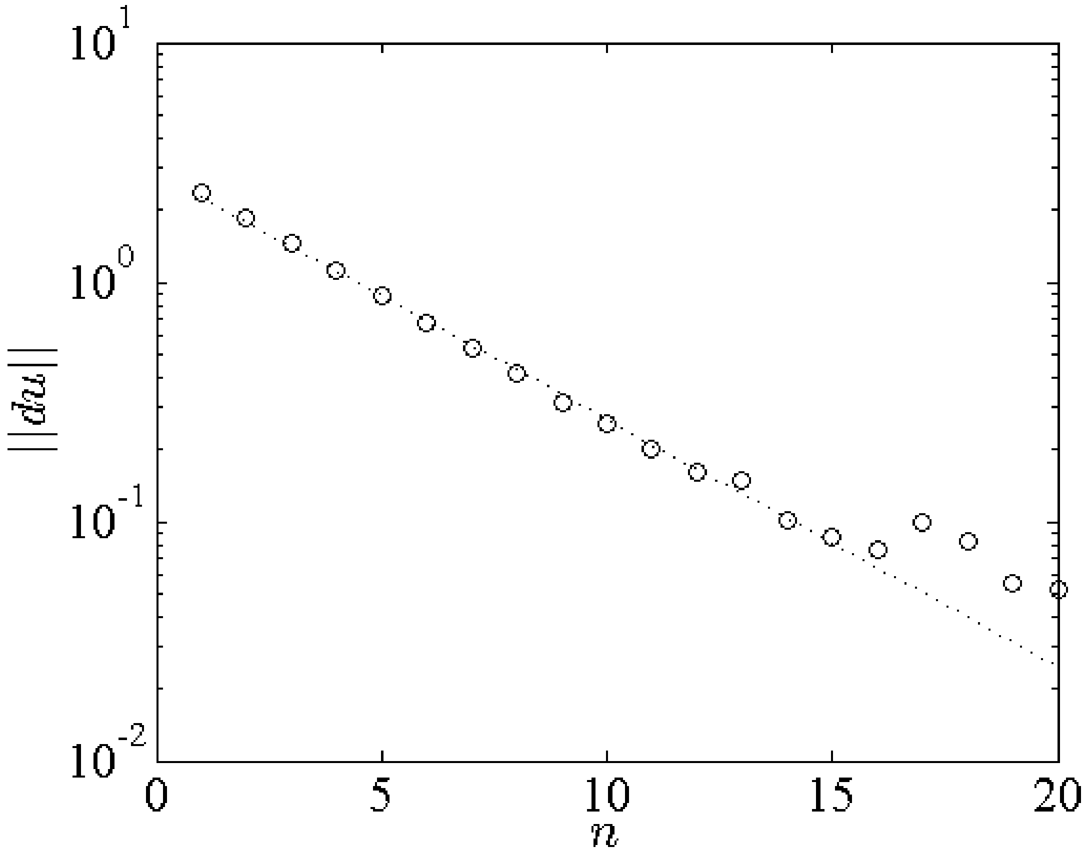
“Reconstruction-free” DVC

- The kinematics can be determined from

$$a = Argmin_b \quad \sum_{\theta} \iint \left(s_2(\boldsymbol{r}, \theta) - \mathcal{P}_{\theta} [\, f_1(\boldsymbol{X} - b_i \boldsymbol{\Psi}_i (\boldsymbol{X}))] \right)^2 \mathrm{d}\boldsymbol{r}$$

- Each increment is determined from a least square fit.

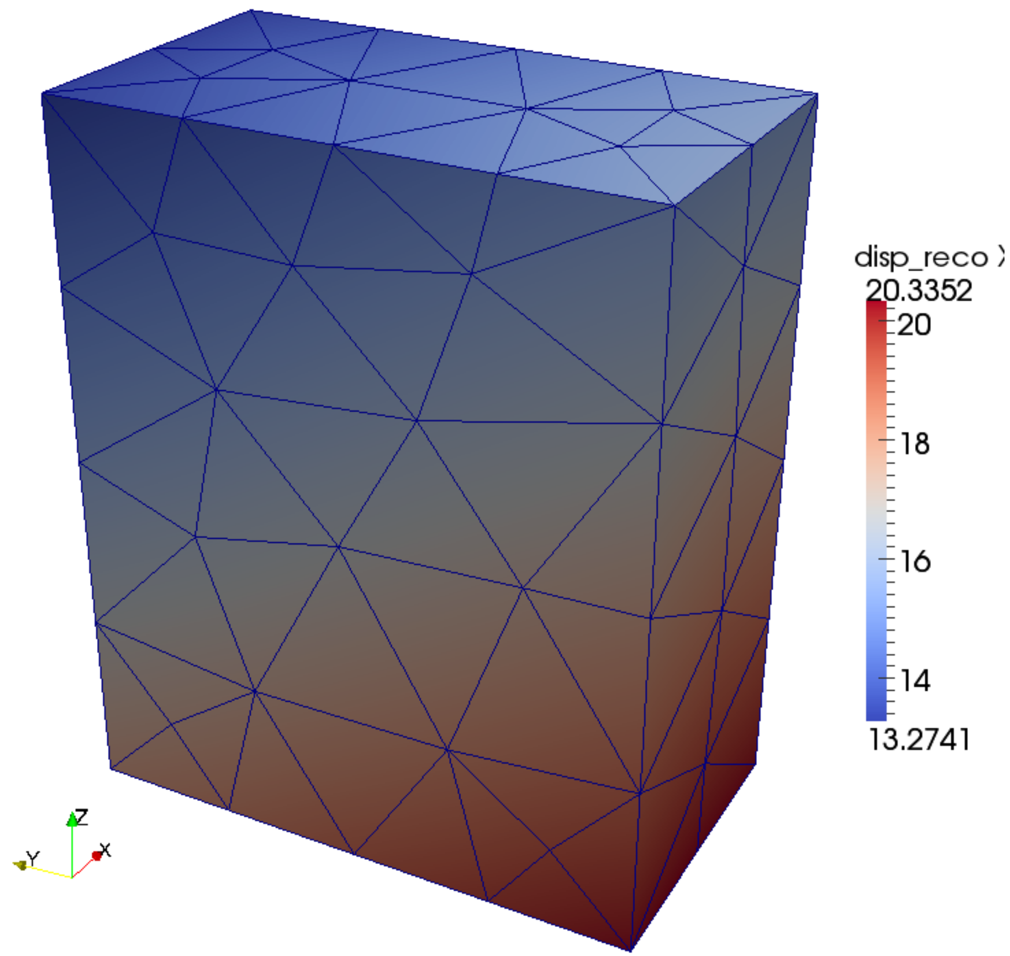
Convergence



Under-relaxation = 0.5

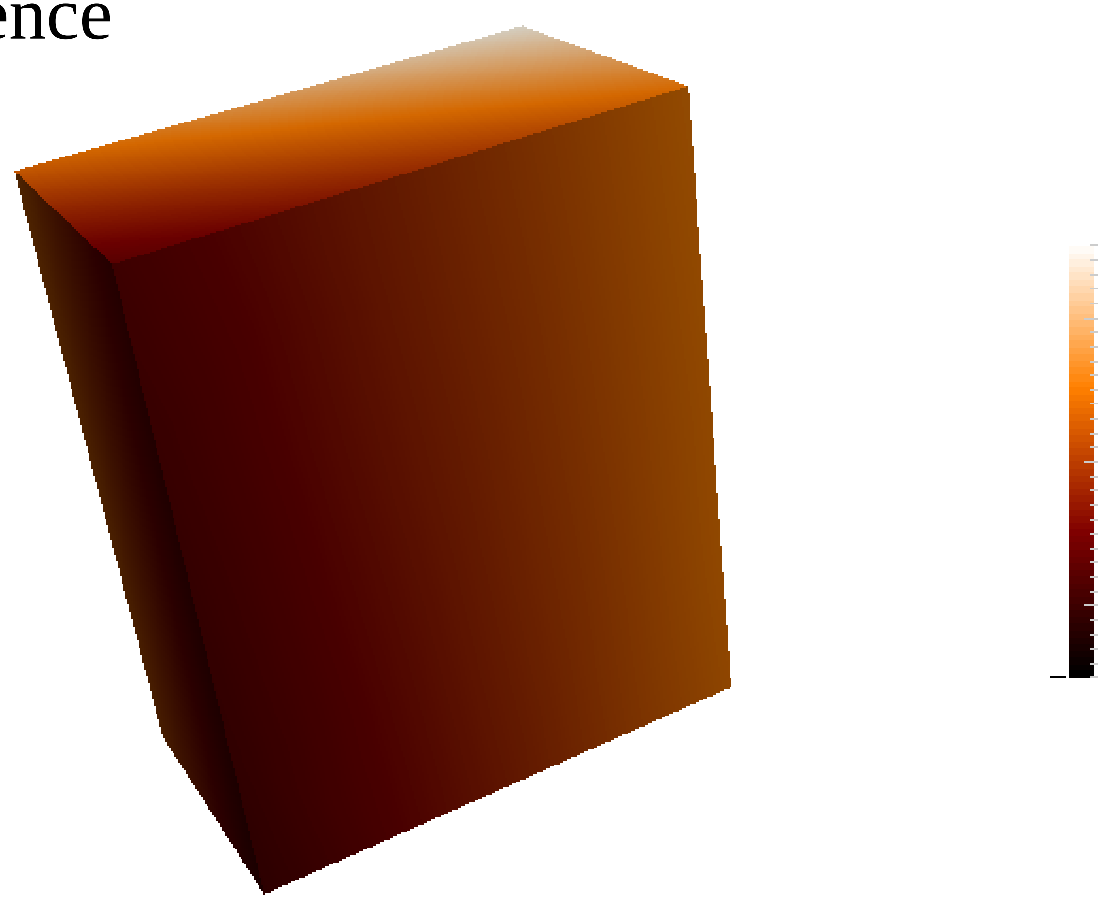
Mesh

- 303 T4 elements
- 97 nodes
- 180×330×400 voxels

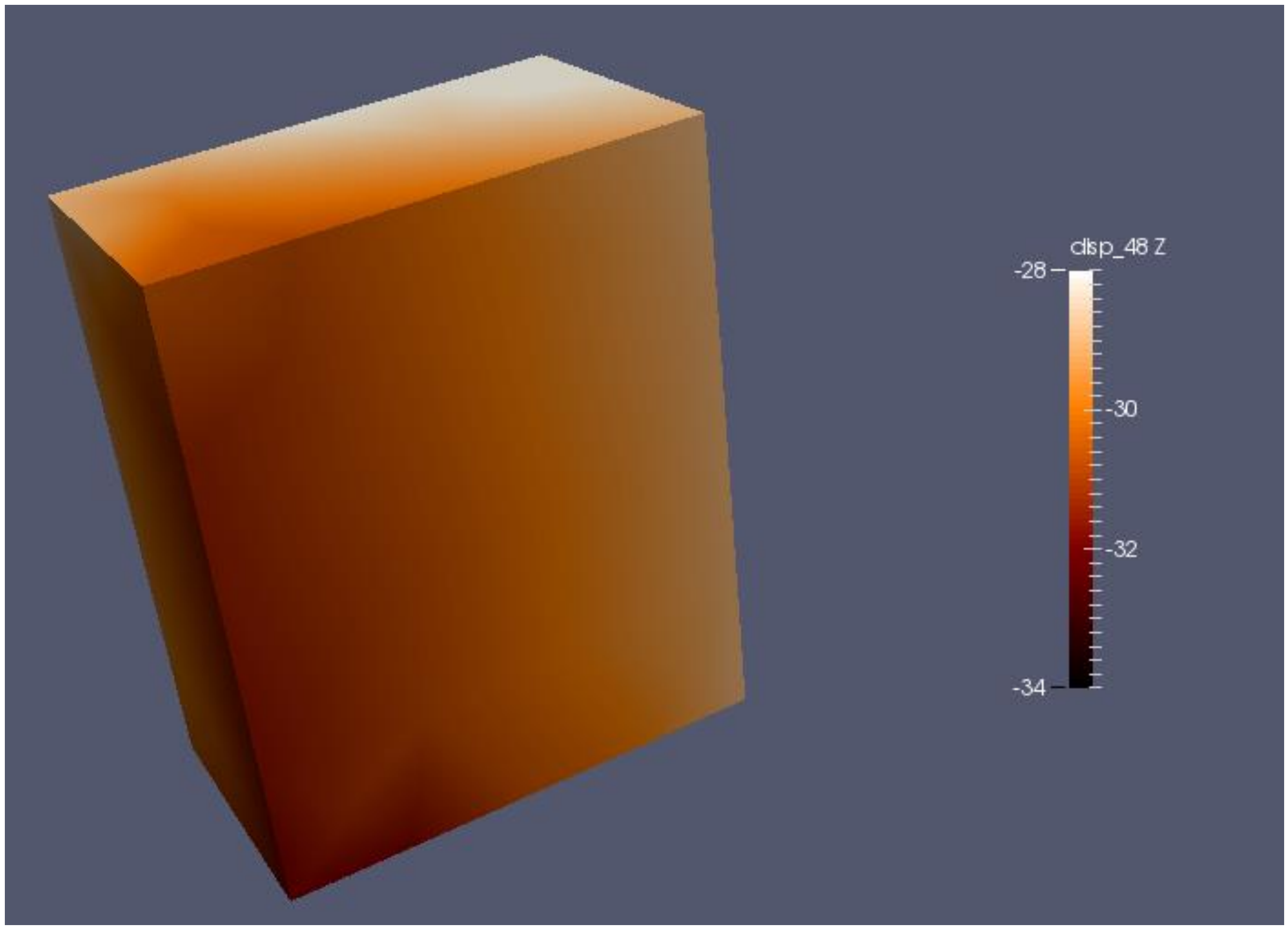


$N_{\mathrm{rad}} = 600$ $\qquad U_z$

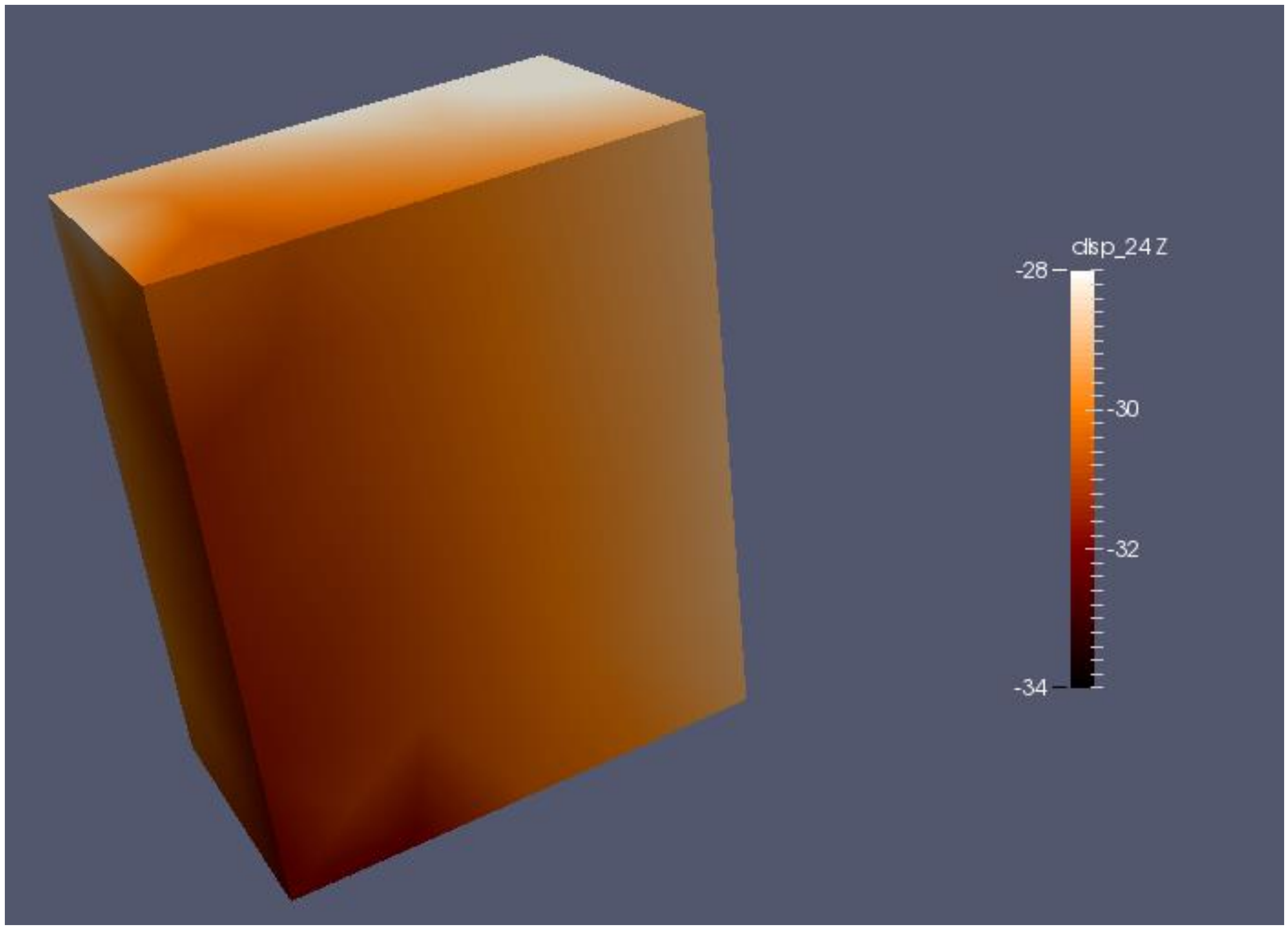
Reference



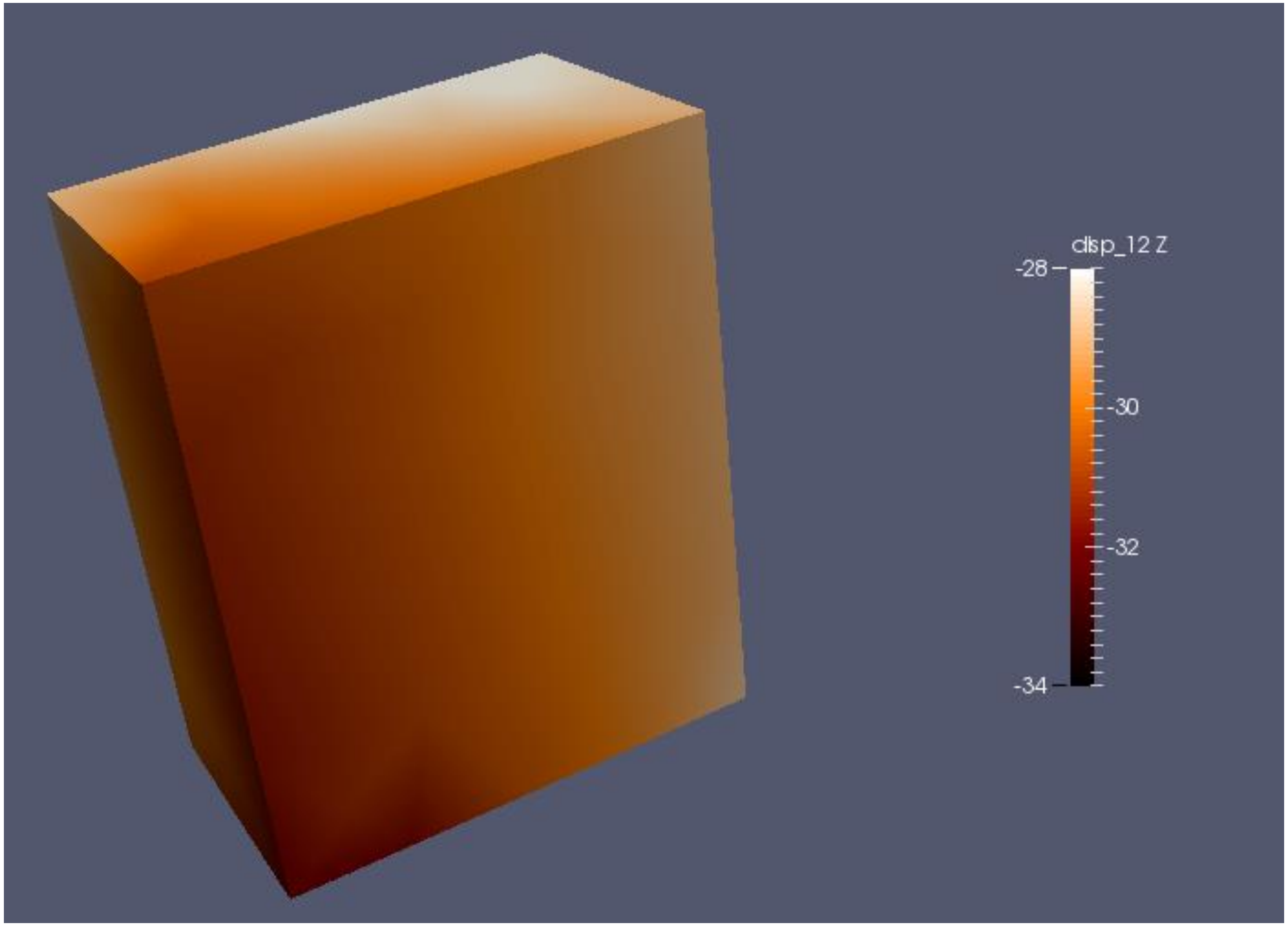
$N_{\mathrm{rad}} = 48$ U_z



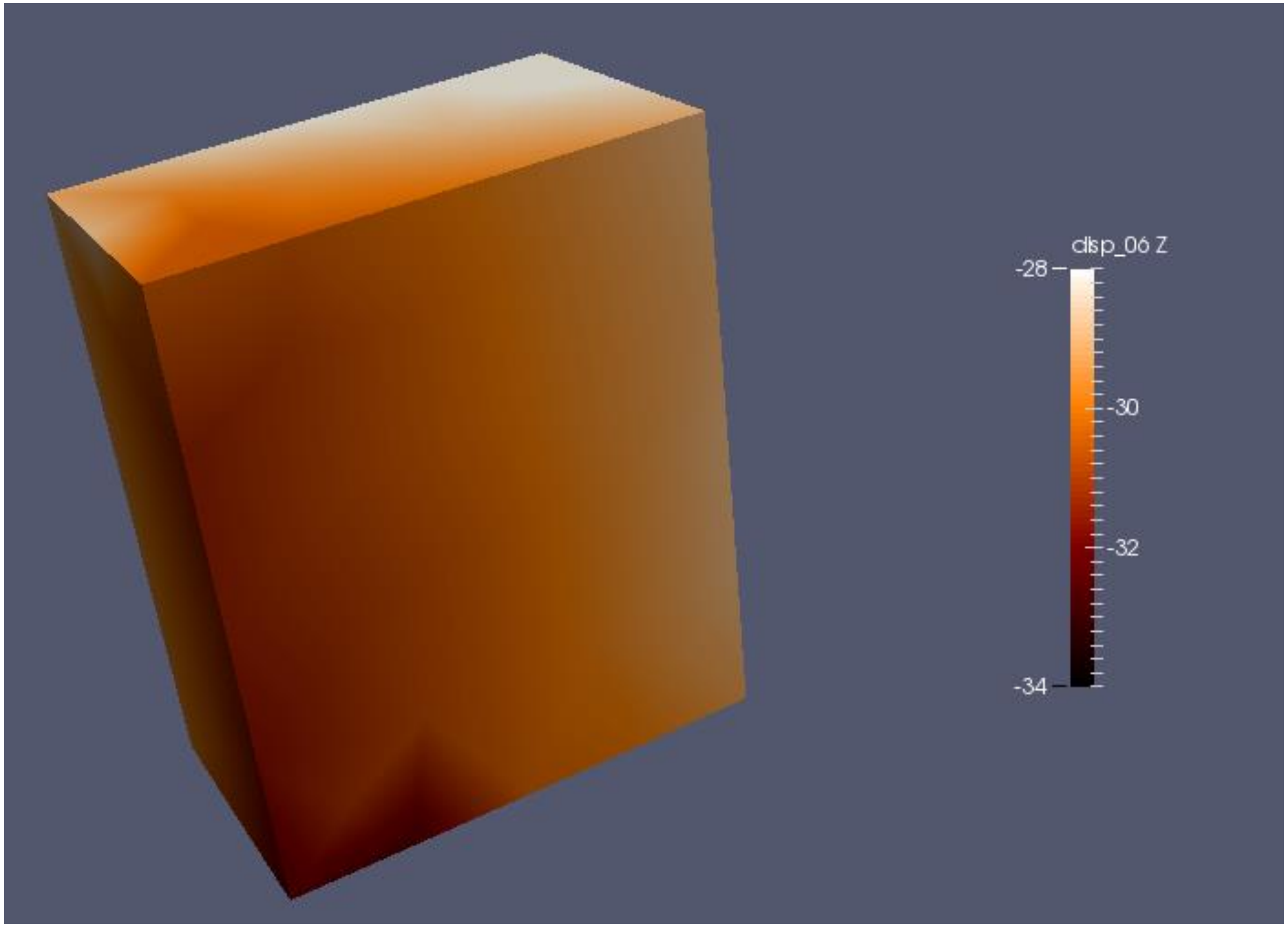
$N_{\mathrm{rad}} = 24 \qquad U_z$



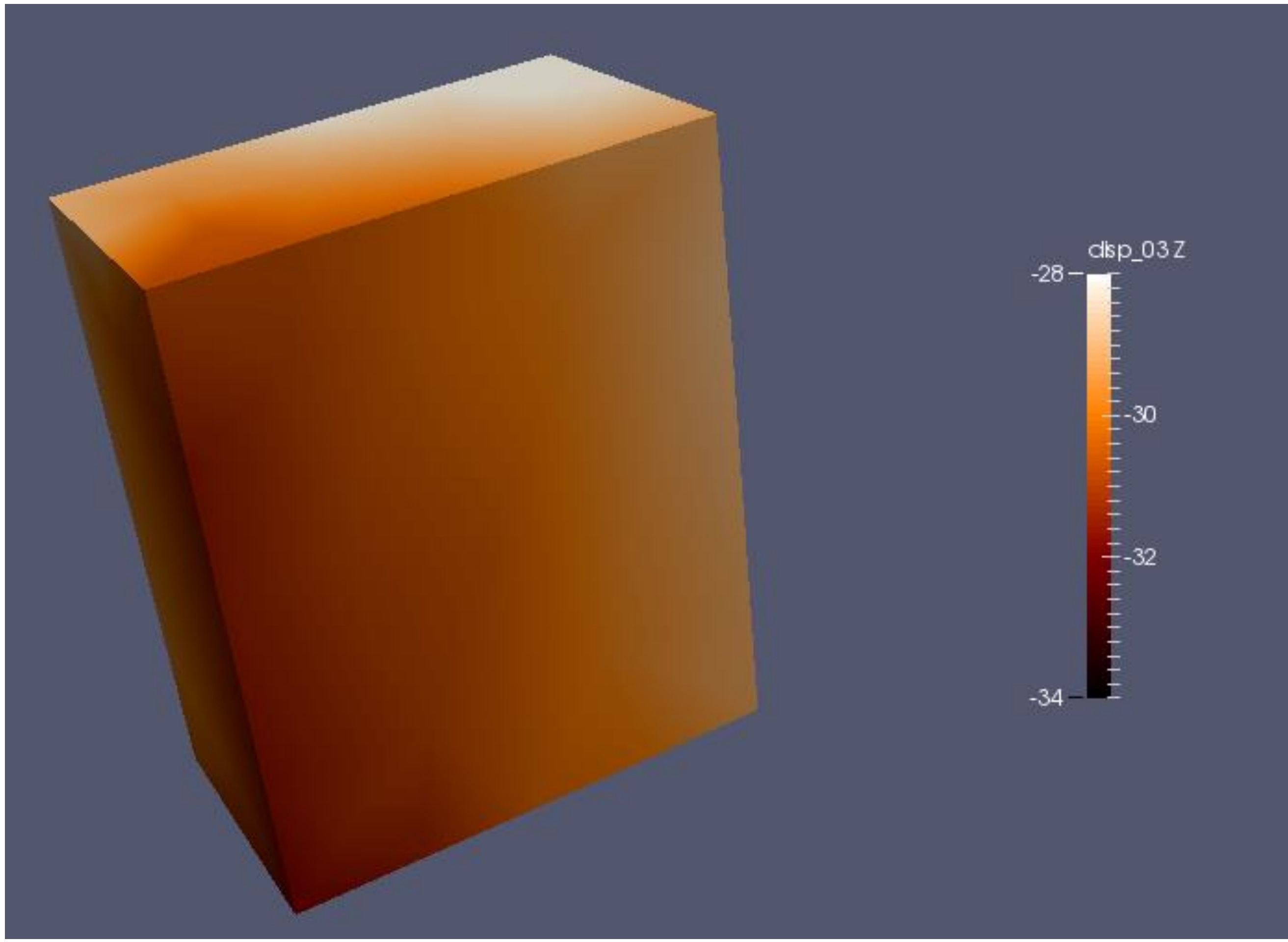
$N_{\text{rad}} = 12$ U_z



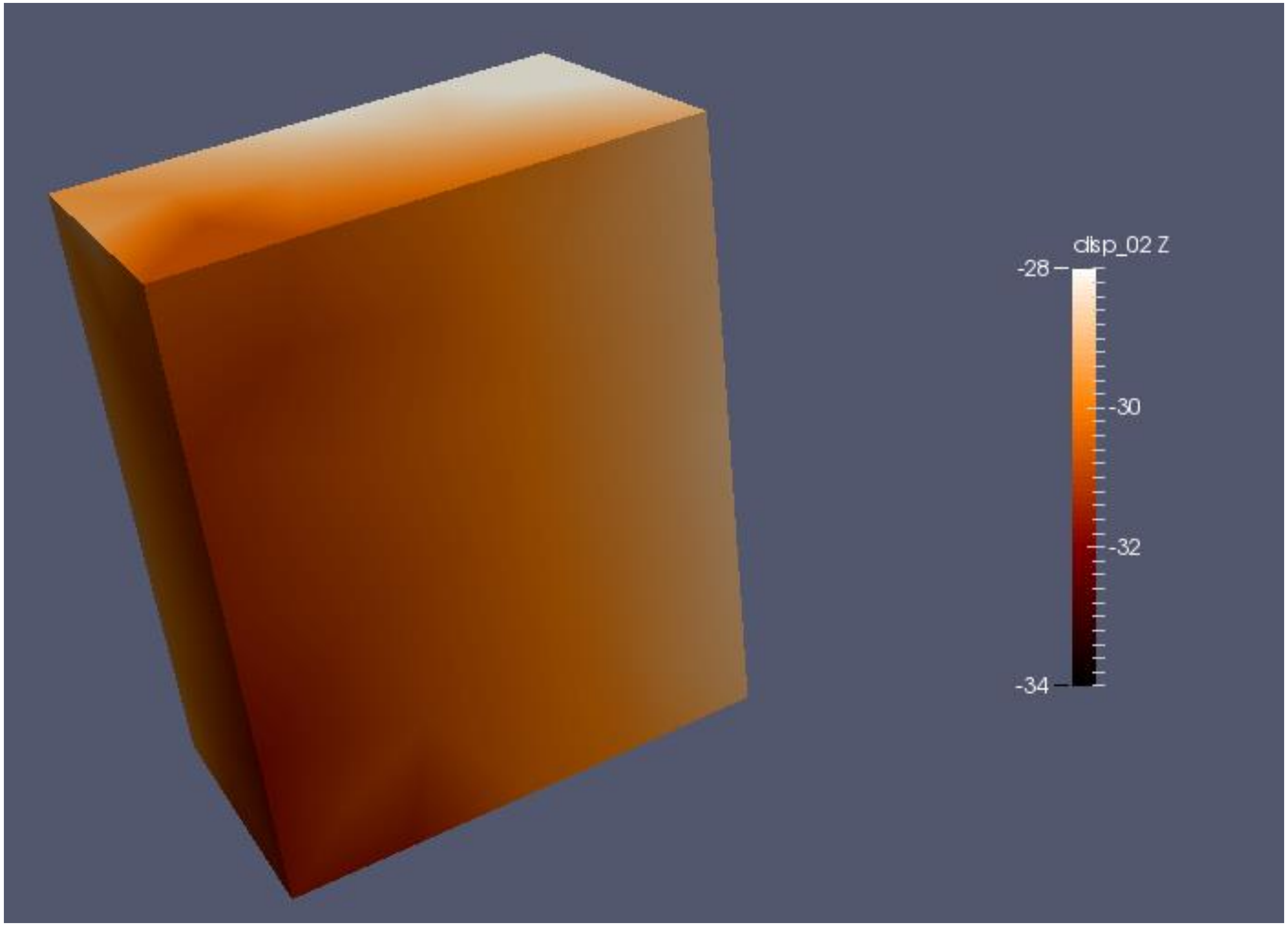
$N_{\mathrm{rad}} = 6$ U_z



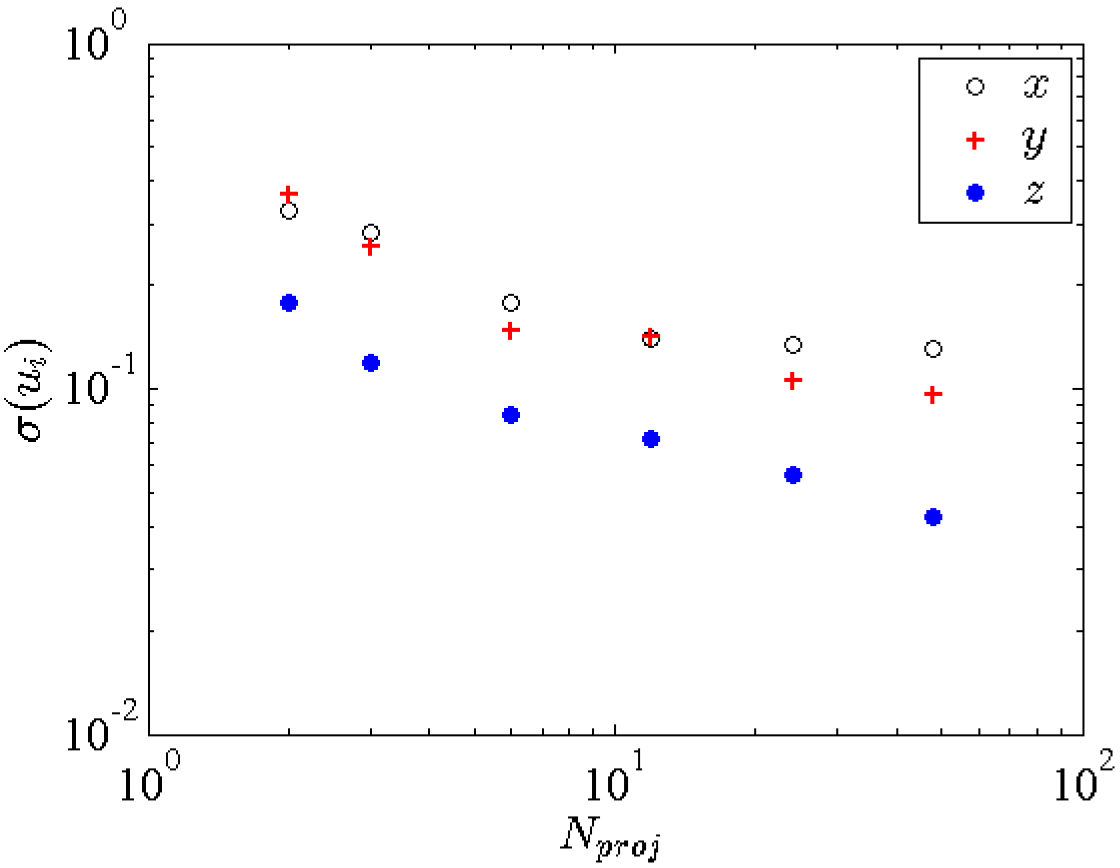
$$N_{\mathrm{rad}} = 3 \qquad U_z$$



$N_{\mathrm{rad}} = 2$ U_z

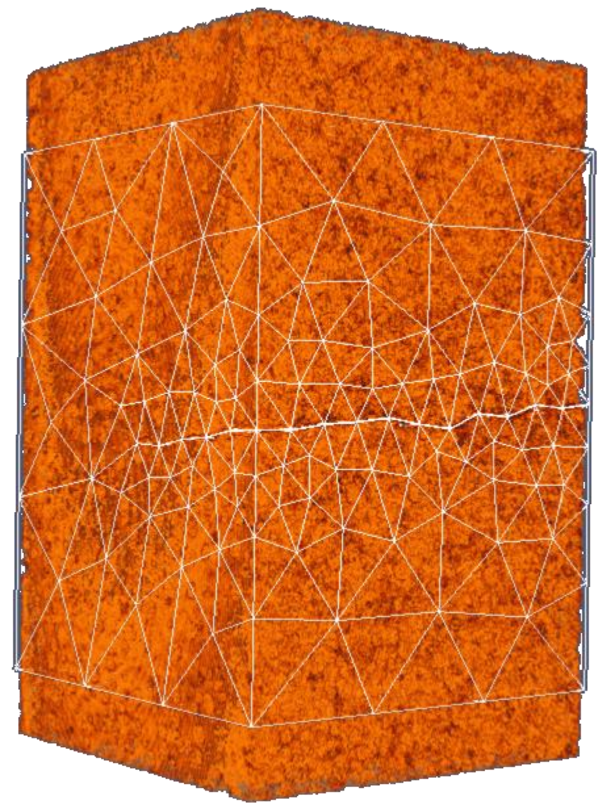


Uncertainty



More complex example

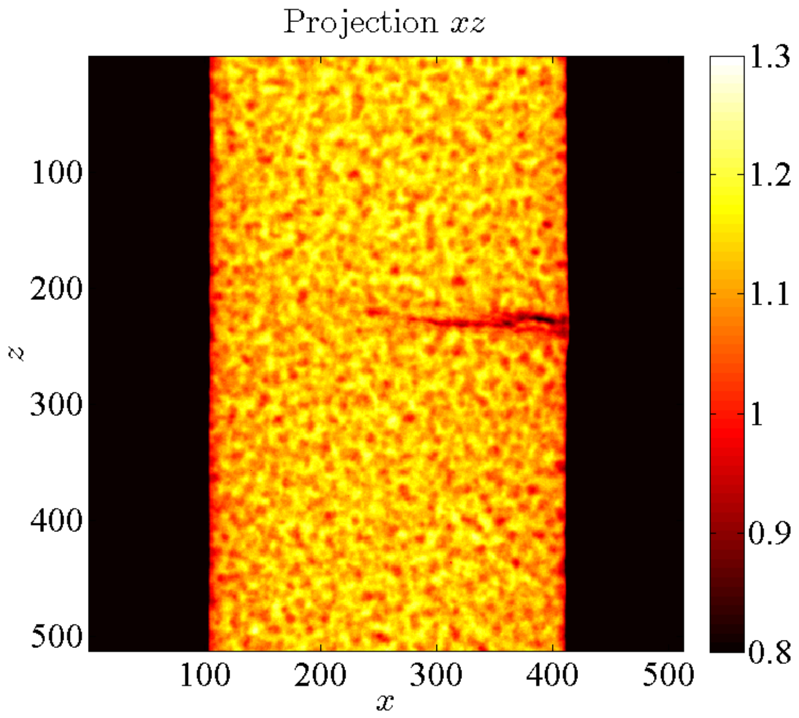
- NG cast iron specimen with a fatigue crack

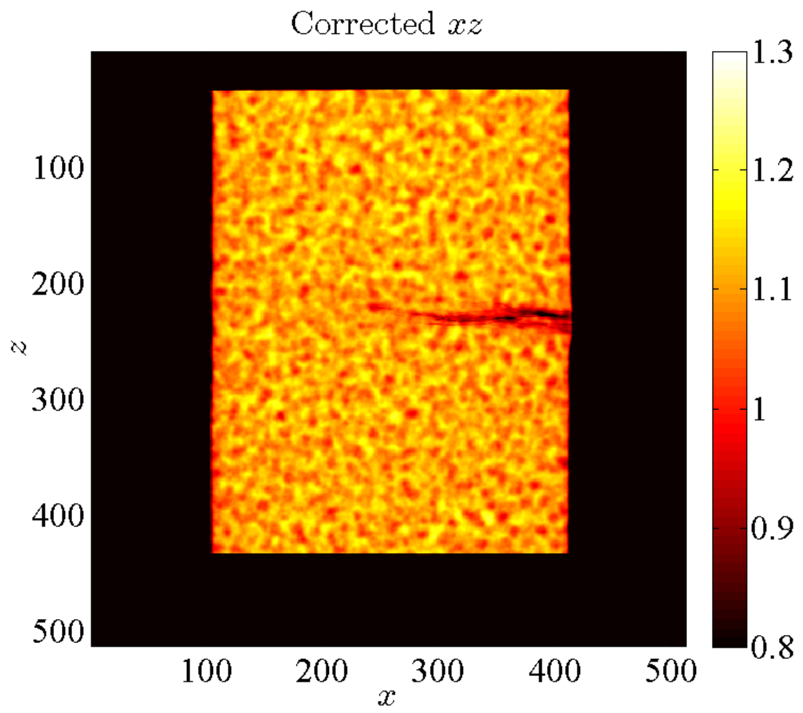


More complex example

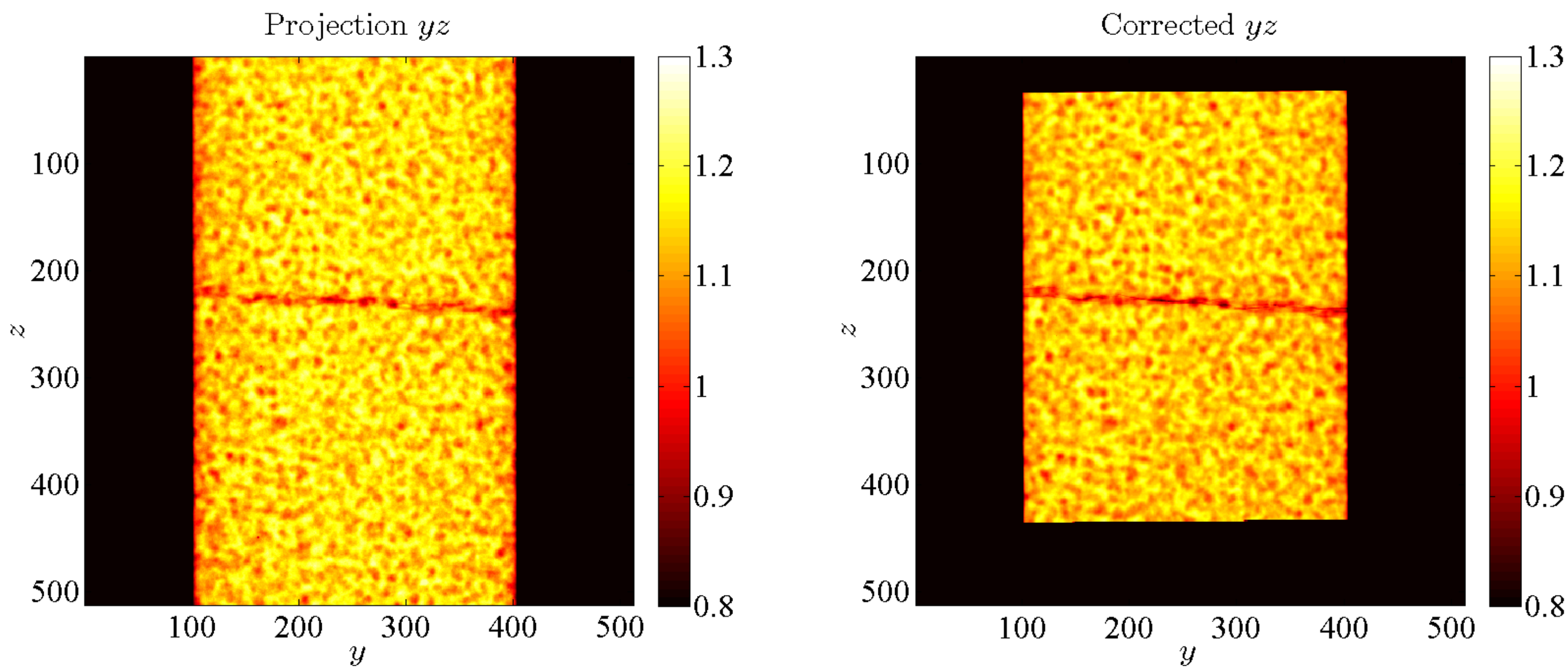
- Analysis of deformed state based on two projections
- Mesh conforming to sample geometry and crack (including roughness)
- Prior assumption: elastic behavior with unknown boundary conditions

Projection 1

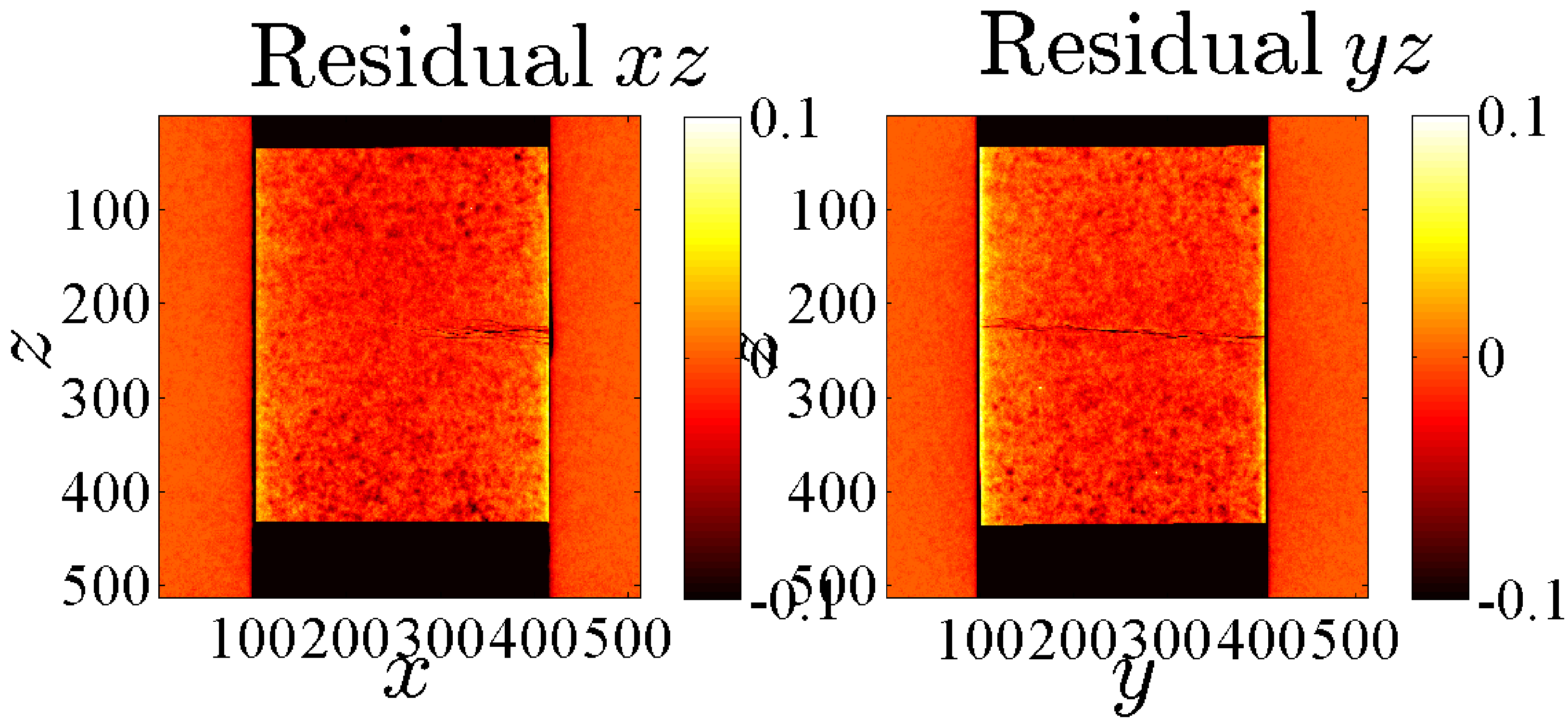




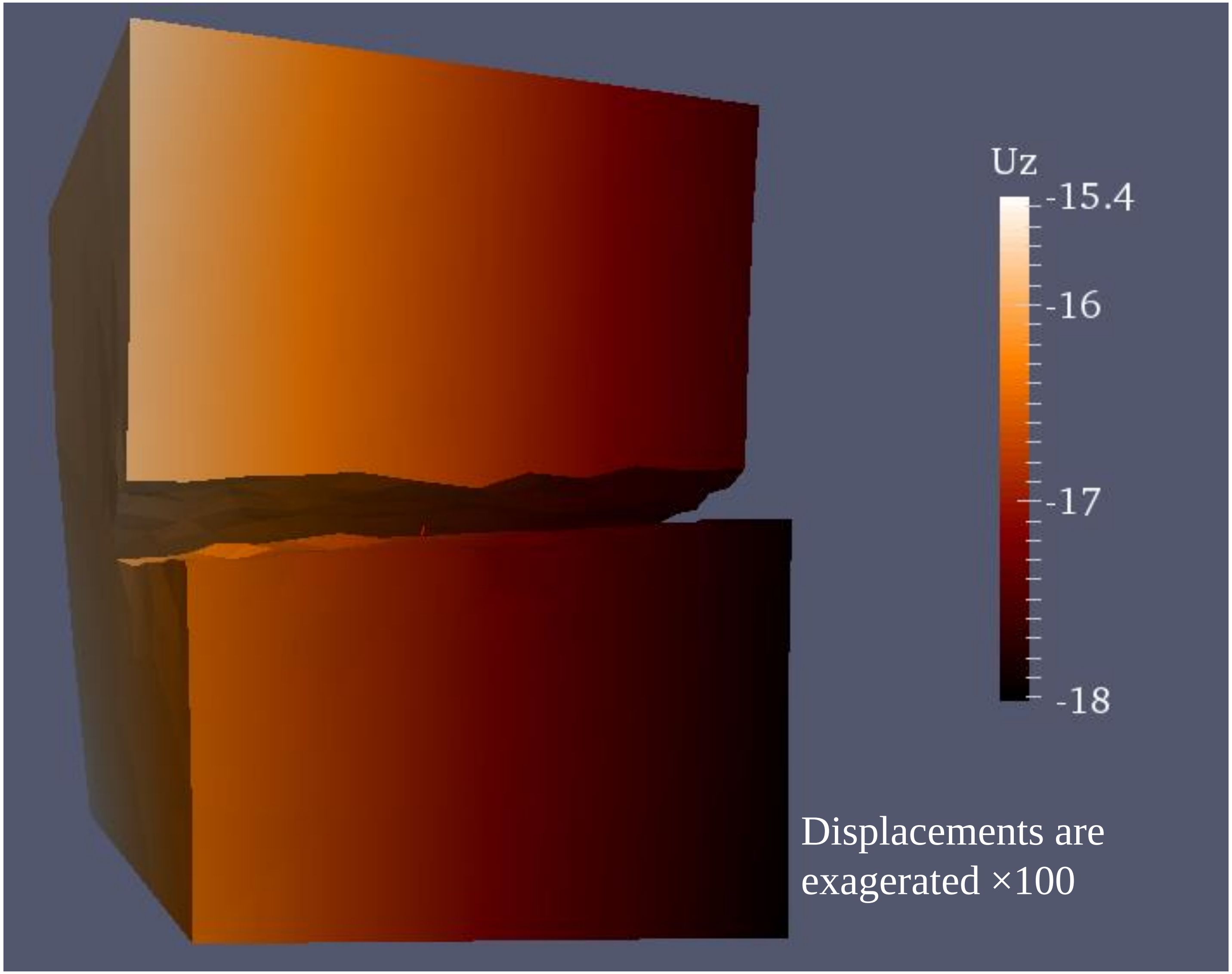
Projection 2



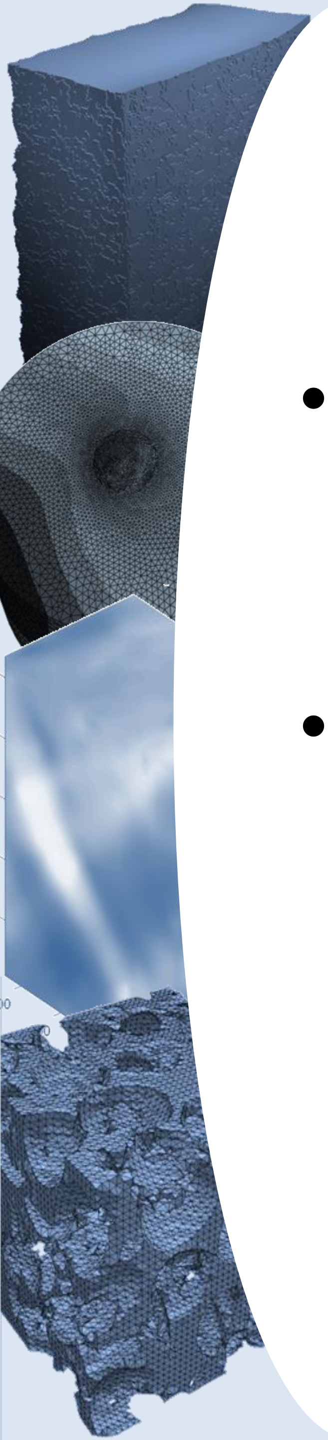
Residuals


Residual xz
Residual yz
0.1
0
-0.1
100
200
300
400
500
z
100200300400500
x
y

Deformed mesh

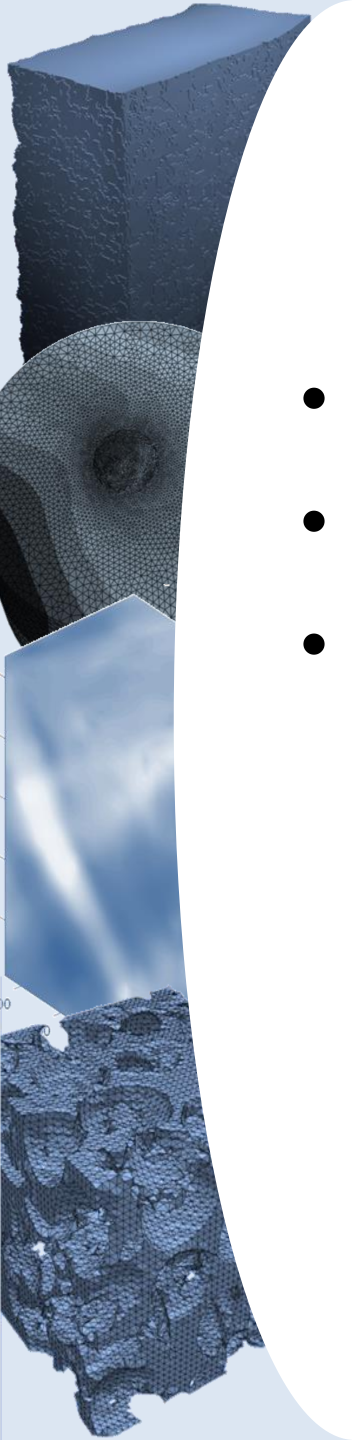


CONCLUSIONS



Conclusions

- Challenge in tomography is to master the "dark" information to extract the most important information
- In both reported case studies, a reduction by two orders of magnitude or more was achieved



Perspectives

- 4D or Dynamic tomography
- Tomography of velocity fields
- Model-based tomographic reconstruction