On the conductivity problem: Ill-posedness and remedies

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Abstract.

Inverse boundary value problems consist of recovering unknown parameters of a partial differential equation (PDE) from boundary data. In applications, this corresponds to reconstructing internal properties of a medium (e.g. conduction, stiffness, density) from observations made at its boundary. In general, parameter estimation problems are highly nonlinear and ill-posed in the sense of Hadamard: small errors in the data may cause incontrollable errors in the unknowns. In view of the many applications, this leads to the search of appropriate methods to contain such instability.

In my talk I will introduce the conductivity problem, a prototypical example of ill-posed nonlinear inverse problem at the basis of Electrical Impedence Tomography (EIT). I will show that by introducing, mathematically suitable but physically relevant, a-priori assumptions on the unknown parameters one can mitigate the ill-posedeness. I will show that for the conductivity problem one can obtain the best possibly stability (i.e. Lipschitz) of the unknowns from the data. These results can then be used to implement efficient reconstruction methods.