About the recontruction of convex lattice sets from horizontal and vertical X-rays

Yan Gerard

We consider the problem of reconstructing a lattice set from its horizontal and vertical X-rays. Without any toplogical or geometrical constraint, it is known since the works of Gale and Ryser in 1957 that the problem can be solved in polynomial time. With constraints of connectivity (such sets are usually called polyominoes) or horizontal + vertical convexity, the problem becomes NP-hard (see Gerhard Woeginger's paper in 2001)

A first polynomial algorithm has been achieved in 1996 by Elena Barcucci, Alberto Del Lungo, Maurice Nivat and Renzo Pinzani for the reconstruction of HV-convex polyominoes from their orthogonal projections. Similar result has been obtained for the so-called Q-convex sets under some assumptions on the sets of directions of Q-convexity. This very nice result due to Sara Brunetti and Alain Daurat in 2003 benefits from Darboux theorem and Richard Gardner and Peter Gritzmann's works about the reconstruction of convex sets. Let us precise that the directions of the Q-convexity should be the directions of the X-rays and satisfy the condition of unicity provided by Gardner and Gritzmann in 1999. It excludes the pair of horizontal and vertical directions. We could use this algorithm to try to reconstruct convex lattice sets - in the classical meaning of digital or discrete geometry, a convex lattice set is equal to the intersection of its convex hull with the lattice - but there would be no guarantee to obtain a convex solution, just a Q-convex one which is a weaker constraint.

It turns out that a basic question remains open : determine the complexity of the reconstruction of a convex lattice set from only its horizontal and vertical X-rays.

The purpose of the talk is to present the difficulties of the problem and in the same time, some things which can be done in the easiest cases.