## HJB equations for stochastic control problems with delay in the control: regularity and optimal feedbacks.

Author: F. Gozzi

Stochastic optimal control problems with delay in the control variable are usually more difficult to treat than the ones with delay in the state variable only. One of the reasons is the fact that, even in the case of control only in the drift, the associated Hamilton-Jacobi-Bellman (HJB) equation does not respect the so-called "structure condition" which allows to apply the methods already developed in the literature. In particular the approach using BSDE's and also the approach of using suitable fixed point arguments in  $C^0$  or  $L^2$  spaces do not work. We present a result of existence of regular solutions for the HJB equations and a consequent result on optimal feedback controls in the case when, as it often occurs in applications, the objective function depends only on the "present" of the state and control variable. The result is based on partial regularization results for the associated Ornstein-Uhlenbeck semigroup (partial smoothing).