

# Maximal regularity for gradient systems with boundary degeneracy

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We consider a general class of elliptic operators  $L$  degenerating at the boundary of a bounded open set  $\mathcal{O} \subset \mathbb{R}^d$  that possess a symmetrizing invariant measure  $\mu$ .

$$L\varphi = \frac{1}{2}\text{Tr} [\sigma\sigma^* D^2\varphi] + \langle b, D\varphi \rangle, \quad \varphi \in C^2(\mathcal{O}), \quad (1)$$

where  $b : \overline{\mathcal{O}} \rightarrow \mathbb{R}^d$  is of class  $C^1$ ,  $\sigma : \overline{\mathcal{O}} \rightarrow L(\mathbb{R}^d)$  is continuous on  $\overline{\mathcal{O}}$ , of class  $C^1(\mathcal{O})$  and such that, setting  $a = \sigma\sigma^*$ ,

$$\det a(x) > 0, \quad \forall x \in \mathcal{O}. \quad (2)$$

These are Kolmogorov operators of diffusion processes living in  $\mathcal{O}$  which are invariant for time reversal.

We show that the corresponding elliptic equation has a unique weak solution in  $L^2(\mathcal{O}, \mu)$  and present some new result for the characterization of the domain of  $L$  under suitable additional conditions.

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