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PATTERNS, WAVES AND CHAOS, AND THEIR REPLICATION IN ACTIVE LATTICES

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ABSTRACT. Pattern formation, wave development (including excitable and oscillatory spirals) as well as space albeit time steady chaos is shown to appear in a (discrete reaction-diffusion) lattice of bistable/’active’ units. Also provided here is a description of the replication process of such forms and waves when using two or more interacting lattices.

1 Introduction

In evolution, from prebiotic to higher levels, replication of form and function are basic processes. Take, for instance, a protein which is already a highly sophisticated form. Crick [1,2] notes that ”at first sight it would seem a very difficult task to make an exact copy of the intact three dimensional structure of a protein in its well-organized native fold. One could conceive making a molecular cast of the surface, as one might for a piece of sculpture, but how would one copy the inside of the molecule? Nature has solved this difficulty with a neat trick. The polypetide chain is synthesized as one extended, rather one-dimensional structure and then folds itself up... The molecule explores the constant opportunities offered by thermal movement until, by trial and error, the best fold is discovered... To produce this miracle of molecular construction all the cell need to do is to string together the amino acids in this correct order ... Life is an infinitely rare event, and yet we see it teeming all around us. How can such rare things be so common?... We need to carry the considerable amount of information as instructions to form the complexity which characterizes life, and unless this information is copied with reasonable accuracy the mechanism will decay under accumulated weight errors. Perfect accuracy is not a requirement. Many of the copying errors will be a handicap but a few are likely to be an improvement... We need these for natural selection to operate on. Thus, we need mutations, as these genetic errors are called".

Replication and mutation (and/or poor fidelity when copying) are necessary requirements for evolution. Is the survival of the "fittest" really needed?

Replication of form seems necessary for the onset of function that belongs to a different synergetic level [3-5]. However, at least in very early stages of evolution, a clearcut separation may not have existed between these two processes. Furthermore, discussion persists, for instance, about which came first, protein sequence or structure [16-17]. On the other hand, biochemical pathways did not evolve by the sequential addition of steps to pathways that became functional only at the end. Instead, they have been rigged up with pieces co-opted from other pathways, hence using nearby accessible raw material. This is the tinkering process advocated by Jacob [18]. A minimum of function (and its replication) may be necessary to have replication of form. But without a significant portion of the latter a minimal function may not emerge, hence form and function may have grown by successive steps of disparate time duration and degree of sophistication. If there is a trial and error search, its efficiency may be drastically improved both in time duration and exploration of landscape.

The (bio)evolutionary problem is indeed quite complex. Take again proteins. the great majority of sequences have multiple steady states and hence may fold into different structures. Faithful replication of 3d forms is indeed crucial. Failure in doing so may lead to inactivity or misfunction. For the simpler case of prions, prion diseases like the Creutzfeldt-Jacob disease in humans or the related ESB in (mad) cows are thought to result from the conformational change of a normal isoform of a prion to a pathogenic form. The cause would be a transition between two alternative steady states of the system exhibiting e.g. bistability properties [14]. Yet, we feel that faithful replication of form in 3D may not be such a rare and difficult event, at an early stage before the (complex) specific function of the 3D architecture is acquired.

2 Pattern formation in a lattice

Bistable units easily yield patterned structure in a lattice. It suffices to suitably adjust the dynamics of the unit relative to the intralattice diffusion. Then at least in systems possessing a variational evolution though this may not be needed for other cases, from a spatially random initial condition we end up in one of the available patterns. For a $N \times N$ lattice with bistable units such space possesses 2^{N^2} states. Fig. 1 illustrates one such possibility. Further details can be found later on in sect. 4 and in Ref. 19.

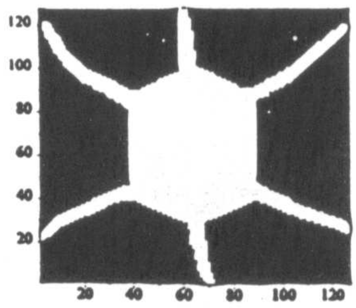

Figure 1: *Pattern/form through mutual interaction of bistable units in a lattice*

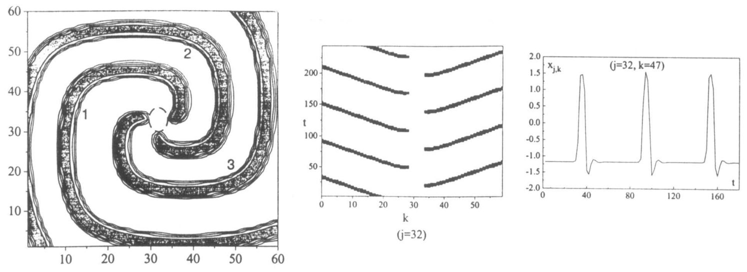

Figure 2: *Fully developed excitable spiral wave in a lattice of Chua's circuits (a) 1-3, snapshots of the wave rotating around its core, (b) space-time diagram of a section of the lattice, (c) (pseudo) oscillations of a unit far away from the spiral core*

3 Wave processes in a lattice

To obtain waves including pulses, spirals and so on, the method again is to suitably tune the dynamics of the unit relative to the intralattice diffusion. We know that if units exhibit limit cycle behavior then global behavior in the lattice in the form of spiral waves is possible. However, with high enough diffusion, hence a fast enough process, we do not need limit cycle behavior of the units to have excitable or oscillating spirals. Indeed, if we have two accessible (bistable) steady states and we appropriately choose the dynamics (kinetics) of the unit, the trajectories may stay for rather long time intervals near each steady state before they eventually decay to one of them. Accordingly, if the diffusion is fast enough the signal emitted by a unit may reach another unit that feels that its neighbour is oscillat-

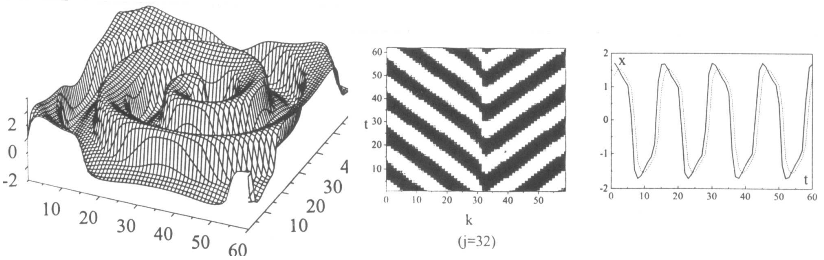

Figure 3: *Fully developed "oscillatory" spiral wave in a lattice of Chua's units. (a) A snapshot of the wave, (b) space-time diagram of a mid-section of the lattice, (c) (pseudo) oscillations of a unit near the spiral core (dashed curve) relative to oscillations of the other units (solid curve)*

ing, though this only occurs for a finite time interval. Thus fast enough diffusion processes relative to slow enough kinetics may very well provide global coherence in the form of wave processes like spirals. Figs. 2 and 3 illustrate spirals constructed in a lattice composed of Chua's units, which a useful electronic circuit with nonlinearity made of broken linear pieces joined together. It offers many possibilities, in particular a bistable mode with two accessible steady states [20].

4 Replication of form

During their time evolution, the interaction between two systems one organized in a given form, a steady pattern, and the other in a disordered state, leads to the former being replicated in the latter, which can be considered as raw material in the initial state. A given form and its replica may be replicated as many times as we wish provided we have nearby enough raw material ready for interaction. Here appears the need of minimum function at the lower level, linking elements of a system, and system to system. Replication produces entities that can themselves be copied by the replication process. This replication process is not merely like that of a printing press.

Let us consider two $N \times N$ identical, interacting lattice systems, irreversibly evolving in time while having accessible a large set of (steady) states. The N^2 reactive units/knots in each lattice, denoted by u and v, for the first and the second lattice, respectively, interact with an intralattice diffusion κ, and an interlattice coupling h. They are individually governed, for instance, by the (FitzHugh-Nagumo) cubic function $f(w) = w(w-1)(a-w), 0 > a > 1$, $(w = u$ or $v)$, hence they are *bistable* units with two (stable) steady states. The use of steady states and taking both systems identical

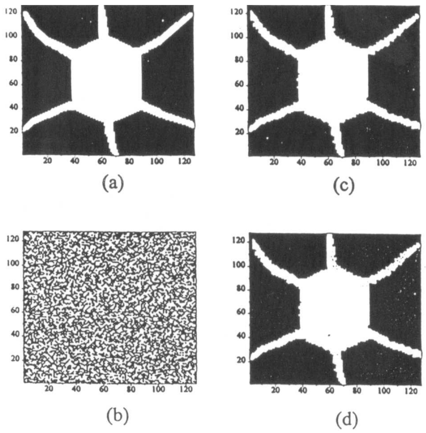

Figure 4: *Replication of form through mutual interaction of a lattice in a state defined by a steady "regular" pattern and another in a state of spatial disorder ("chaos") or raw material, (a),(b): initial conditions, (c), (d): off-spring of two identical patterns.* $a = 0.5, \kappa = 0.006$, *and* $h = 0.4$.

greatly simplified the task. Yet the huge space of (2^{N^2}) accessible states makes the interaction and evolutionary problem highly nontrivial. Besides, using a lattice system makes a (loose) connection with a ”primaeval soup”. Due to the form of $f(w)$, the units, or raw materials, are significantly complex and, moreover, are ready for interaction and replication. Then for the parameters, a, and h, taken in a suitable region the two lattices may synchronize, hence replication occurs [21, 22].

For illustration take Figs. 1 and 4 where one lattice initially possesses a given form (Fig. 4a) and the other is the raw material (Fig. 4b). As time proceeds and the mutual interaction operates, the two systems evolve until they produce identical patterns (Fig. 4c, d). replication for a=0.5 is quite faithful. This is not so for a≠ 0.5, though we have control of the possible and actual degree of fidelity. A serious shortcoming of the work so far done [19,21] is that there is no ”natural” selection mechanism. Rather when two patterns compete, a mixed combination is produced in duplicate. It is only when one of the two systems is in a spatially ”chaotic” state that a form dominates, and, as the winner, it is indeed replicated.

5 Wave replication

As expected, transfer of wave processes (and pulses, etc) in a lattice to another lattice is also possible by suitably adjusting the relationship between intra and interlattice diffusion and the dynamics (kinematics) of the units.

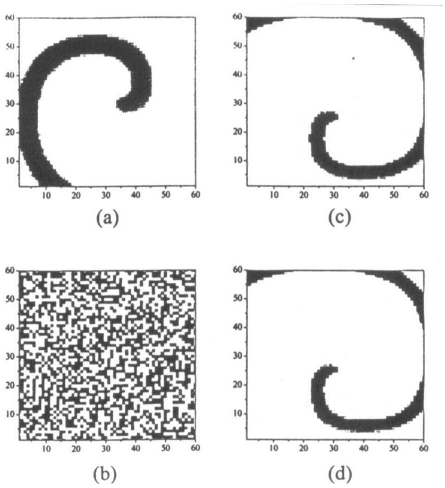

Figure 5: *Replication of a spiral wave (a) initial spiral wave in a first lattice, (b) initially space chaotic state in a second lattice, (c), (d), synchronized spiral-like state or replication of spiral wave behavior.*

Fig. 5 illustrates this replication process [23].

6 Conclusion

As said by various authors, in science, metaphor spawns theory, the ultimate value of which is judged by comparison with experiment. Metaphor also plays another role, as a vehicle for carrying powerful concepts and images from one area of science to another [24-29].

Note that in the illustrated replication processes neither catalyst agents have been introduced nor any statistics (thermal movement) is involved, other than that associated to the choice, or spontaneous appearance, of initial conditions.

Ideologically, there is yet no owerhelming evidence for any and theory of replication of form in complex systems. Nonetheless, there is sufficiently strong argument for the scenario of tinkering [18] in the realm of unsteady, metastable, long lasting structures to push for its development. Indeed, the life of those who bear, say, genes is transient. Even genes do not last for ever. What (bio)evolution seems to demand is opportunities (including a long enough life time) with available appropriate raw materials to replicate on. The Darwinian survival of the ”fittest”, when only steady states, permanent forms, are considered appears to me as too conservative an attitude. Long lasting metastable forms, having significant function, may be enough to win, be replicated and so on.

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