ERNEST MALINVAUD[*]
CREST - Dept. de la Recherche

MATHEMATICS AND DEVELOPMENT OF ECONOMICS

Conferenza tenuta 14 ottobre 1997

Introduction

Propositions to be first argued here can be summarized in just one sentence: the role of mathematics was important in the development of economics but non longer appears to be critical. This role is now taken for granted, which was not the case when the Seminario Matematico e Fisico di Milano was instituted, nor even around 1950. There will be no retreat in the recognition of the usefulness of mathematical modeling for economics. Today the question non longer is it know whether to use mathematics, but how to make proper use of it in economic research, in the teaching of economic theory, in academic selections or promotions.

A large number of mathematical problems can be posed in economic research and some of them are really thorny. But progress in economic knowledge is not dependent on solution of all these problems to the same degree. In some cases it is so little depends on it that, even if the solution is found, it is soon forgotten.

The mathematical model of a theory and the properties of this model are only a part of the theory. If the rest is ignored, meaning, relevance and applicability of the theory are misunderstood. Unfortunately, teaching nowadays focuses so much on the models that they often happen to be taken as the unique ultimate goal of economic learning.

Too often also professors of economics are appointed and promoted now with reference to a main dominant criterion, the mathematical expertise and sophistication they exhibited in their published work.

---

[*] Contribution to the seventieth anniversary study day of the Seminario Matematico e Fisico di Milano (Università degli Studi and Politecnico), October 14, 1997

Such insatisfactions about misplaced research, inadequate teaching or inappropriate evaluation are worth mentioning as side issues, because they concern mathematics in economics. But they do not belong to the core of this paper, which should aim at explaining why and how mathematics entered economics, also at presenting types of questions for which mathematical research was involved in economic research. Insatisfactions will then non longer be mentioned in the three parts of this paper, which will respectively concern: (i) fundamental reasons for the mathematization of economics, (ii) history of the phenomenon, (iii) interfaces between economic research and mathematics.

1 Why had economic theory to become mathematical and economic research to often use mathematics

(1) Economic phenomena concern large and complex societies, in which individuals and firms specialize as for the work they perform or the output they produce. This specialization, which leads to high productivity, requires a large variety of exchanges so that firms get the input they need and individuals the goods and services they want. Exchanges are made against money at prices that are often bargained but under strict limitations imposed by market opportunities. These operations also involve complex infrastructures and services, which are provided by government, as well as by financial institutions. The outcome of these many operations may be more or less efficient, resulting in various levels of standards of living, various degrees of employment and use of productive capacities, various trends in inflation or appreciation of assets. So, economic activity is more or less gratifying with respect to individual and social objectives.

The aim of economics is, of course, to understand these phenomena and to propose actions likely to improve upon observed performances or to cure malfunctionings such as mass unemployment and high inflation. Economic theories encapsulate this understanding and provide frameworks for finding appropriate actions. Notice that the word ”theory” must here be meant in a wide sense: a theory concern a phenomenon or a problem; it provides a methodical intellectual constructs, of a synthetic nature, for the knowledge and analysis of the phenomenon or problem; this construct aims at being appropriate within a domain of some generality, subject to specified hypotheses: definition of the domain and, within it, correspondence between the intellectual construct and reality belong to the definition of the theory. A science is made up of more or less tightly interrelated theories; in economics the interrelations are not very tight, more so than in other social sciences but less than in developed natural sciences.

It happens in some economic theories that their intellectual constructs can be fully given in ordinary language, with the required clarity, accuracy and rigor. But in more and more cases, dimensions and complexity are such as to forbid that. At the center of the intellectual construct a mathematical model has then to be defined, so that the assumed relations between the concepts are clearly exhibited and lend themselves to logical reasoning, as well as to empirical validation. Fundamentally there is no more to say about the reason for the mathematization of economic theories: mathematics provides the appropriate language for dealing with central parts in these theories, parts that would remain too vague if their expression was bound to use only ordinary language. Notice, however, that the latter is not expelled; it is still indispensable for discussing the many aspects of the correspondence with the real world: in the definition of concepts and hypotheses, in the interpretation of results. I already alluded to this fact when complaining about a frequent deficiency of economic teaching in this respect.

(2) However, for our purpose here, it is important to have in mind how the appropriate mathematical models are found in economics, or equivalently how the appropriate economic theories are built. They must, of course, come from analysis of the phenomenon or problem and from prior knowledge relevant to it. But where does this prior knowledge come from? Not from the same kind of information as in the natural sciences, and this has many implications, including on the kinds of mathematical problems we economists have to solve,

In the natural sciences knowledge mainly comes from the observed results of controlled experiments. Such experiments are very rare in economics, although a movement promoting them has recently shown their usefulness in some context. Most often our statistical observation is passive, in the sense that it bears on the results of economic activity as it happens to be; since phenomena are complex, a multiplicity of causes is involved and even important ones many have escaped the attention of research workers. In order to tackle this difficulty, we need to simultaneously estimate the effects of all identified causes.

With respect to the challenge, economic data bases are rather poor in most cases. Time series for a country are short, either because statistics are not available for earlier periods, or because the economist has good reasons to fear that the phenomenon under study has changed since then. Cross-sections of countries or regions may nowadays contain more observations, but the number of potential causes of variation is also larger. Cross-sections of data on individuals, household or firms often now bear on thousands of units, but again heterogeneity is large among them, requiring introduction of many incidental variables whose effects must be simultaneously estimated; moreover, aggregating relations observed between units in order to derive laws of global phenomena is delicate.

In order to build scientific knowledge form such ”outside observation” econo-mists are therefore very handicapped in comparison with their colleagues working on natural sciences. Fortunately however, economists have some inside knowledge of their domain of investigation. Institutions in which economic activity operates were consciously built with given features and for given purposes. Individuals are human beings, whose economic motivations are fairly well known. Firms were created by those who thought that their operations could earn a profit, or at least could be self-financing in the long run. All this inside knowledge is valuable for those who try to understand and characterize the global consequences of economic activity.

This is why the efficient methodology in economics most often consists in combining inside microeconomic knowledge with statistical observations on either individual behavior or global phenomena. Although it is a simplification, we may say that a model is built from the inside knowledge, that the model then contains unknown parameters. The simplification involves in particular a short cut, because the model specified in a particular research project has a good chance to proceed from a prior knowledge that results not only from inside knowledge but also from the knowledge earlier obtained thanks to the processing of some data.

(3) Given this methodology, it is no surprise to learn that economists often refer to ”the probability approach”, which in all fields applies to induction from statistical data: tests and estimations are made within stochastic models defined in such a way as to represent the generation of the data and to imply in particular the phenomenon under study. It is no surprise either to see that an important part in the set of mathematical models used by economists is made of the stochastic models serving inductive purposes, the so-called econometric models. An important part of the mathematical problems of economics correspondingly concerns definition and properties of appropriate inductive procedures.

For specialists of natural sciences it may be more surprising to learn that an important part of economic theory was built, and still is being built, without reference to outside observation but only from the inside knowledge. Formalizing the known institutional framework of economic activity and the rationality of economic agents in the pursuit of their objectives leads to models which have significant properties, subject to sets of hypotheses that are important to know.

Most mathematical economists would even argue that mathematical research on these models formalizing direct knowledge is more important in economics than mathematical research on inductive procedures suited to econometric models. One of their arguments might be that the first kind of research started much earlier than the second. History, to which we are now turning, indeed supports the argument.

2 How did mathematics enter economics

Introduction and diffusion of the mathematical language into the discipline of economics was due to many scientists, not all of them qualifying as ”mathematicians”, not even all well-equipped with mathematical competence. It does not matter much here, where the subject is the role of mathematics not that of mathematicians. A few names will be mentioned, because they marked the history of the process, but no attempt will be gauge whether they contributed to mathematics. Anyway, the history will be cursory and distinguish just three phases: the century before the first world war, the interwar period, the second half of this century. A few words will be added on an incidental issue, about which I often read mistaken statements: the role of physics in economics.

(1) From around 1830 on, most great theorists of economics realized that mathematics would help them. Karl Marx was no exception, although he was not really trained to think in mathematical term. Progressively in earlier times precise abstractions had been introduced for the elaboration and diffusion of theories. A natural role existed for the mathematical language.

The first significant case was the mathematician Antoine Augustin Cournot (1801-77), best known in economics for his early formalization of interactive rationality; this was in the study of duopoly (on a market there are just two suppliers), a natural step after monopoly theory. The solution defined by Cournot was later the subject of a controversy involving another mathematicians, Joseph Bertrand (1822-1900).

The theory of the general economic equilibrium still more required mathematical formalization and was indeed sometimes identified with ”mathematical economics”. The list of those who contributed to its development is long, even if we limit the horizon to the first world war. For the purpose of this paper I shall just quote four names: the French economist Léon Walras (1834-1910), the Austrian economist Carl Menger (1845-1921), the English mathematicians Francis Edgeworth (1845-1926), the Italian Vilfredo Pareto (1848-1923) who, after being and engineer, wrote and taught on economics and sociology.

But it would be wrong to assign a specific field in economics to the early introduction of mathematics. The Swede Knut Wicksell (1851-1926) so studied public economics and monetary economics. The American Irving Fisher (1867-1947) clarified capital and income accounting, investigated intertemporal choices and contributed to the study of business cycles.

(2) Notwithstanding so many theoretical advances which we now highly regard, a large majority of economists kept disparaging the use of mathematics in their discipline, like Pierre-Paul Leroy-Beaulieu (1843-1916) who did so at the Collège de France up to his death. In the interwar period, ideas began to evolve; but academic teaching of economics remained almost ev-

erywhere literary.

A number of forerunners then realized that popularization of mathematics among economics was needed. They started writing on ”mathematics for economists”. The phrase is, I believe, characteristic of the interwar period. It fostered a double ambiguity.

First, it conveyed the idea that economists could easily learn whatever mathematics they needed: some tools were available and permitted to solve economic problems. Books were indeed rather elementary, selecting easily examples, drawn from the economic literature or made in order to show how to apply a piece of formal theory. I believe that books sold well; I suppose they served in showing to students they should no longer be resistant to mathematics if they wanted to succeed in economics; but I should be surprised if an investigation would show that the books really helped older economists.

Second, the phrase also conveyed the idea of a part of mathematics particularly suited to economics. The idea is not completely false, if we add that this part cannot be easily identified in advance. Developments in economic theory or econometric methods may lead to new problems belonging to parts of mathematics so far untapped by economists. This indeed occurred in a number of cases and will keep occurring in the future.

Overall, the movement promoting mathematics for economists underestima-ted the importance of the challenge. It did not show the great variety of formalizations and problems that human abstraction can detect; it did not explain how long and strainous has a mathematical education to be if it should well equip for all types of deductions and for knowing how to put imaginative ideas to work. However, it prepared the mathematization that was going to take place in economics on a grand scale during the third quarter of the century. also important in this last respect was the creation, in 1930, of the Econometric Society, gathering in its beginning just a small group of mathematical economists and econometricians, but ready to promote high standard and to quickly expand two decades later.
3) The mathematization was measured by Gérard Debreu [†] thanks to a few indicators. it is impressive. The leading periodicals in the field of mathematical economics published a number of pages that doubled every nine years between 1944 and 1977. In the American Economic Review, taken as an example of journals in which good academic economists publish, the proportion of refereed pages including mathematical expressions went from 3 percent in 1940 to 40 percent in 1990, and the expressions became more elaborate. In the 13 US departments of economics which were labeled as ”distinguished” or ”strong” according to the scholarly” quality of their faculties, the proportion of professors who where fellows of the Econometric Society was less than 1 percent in 1940 and close to 50 percent in

---

[†] G. Debreu, "The mathematization of economic theory", American Economic Review, vol. 81 (1991), p. 1-7

1990.

It may, however, be argued that the process was achieved, for most practical purposes, around 1980. By then the use of mathematics was established in all branches of economics where knowledge was advanced enough to permit reliable formalizations, or where well defined problems were posed, or still where the need of quantification was felt. What remained unaffected was peripherical, descriptive, qualitative or bound to be long heuristic.

The actual use of mathematics at the end of this process appears uneven, both as to the parts used and as to the intensity of use. Giving a detailed account would be long and take us too far here. In the last part of this talk I shall select a sample of hard cases, in which some of the needed mathematical properties had to be found in order to allow completion of economic research projects. This will hopefully convey a correct vision of the situation in the most mathematically advanced branches of my discipline. The rest does not need to concern this paper, except for an additional remark.

Mathematics has become an instrument of economic teaching. In order to learn economic theories students have to familiarize themselves with models of these theories, they do so thanks to exercises, for the solution of which mathematical properties of the models are important and adequacy to the economic subject is less so. In order to learn how to infer from data, they have not only to deal with statistics but also to study the formal properties of estimations or tests in the particular cases with which they are confronted. Many students, especially the less gifted, have the feeling that they are taught mathematics rather than economics. It seems to met that this situation is similar to what happened in the teaching of physics and many be similarly justified.

(4) Since the Seminario concerns physics as well as mathematics, I am taking this opportunity to dismiss a thesis that was, and still is, often read about mathematical economics, namely that its proponents were aiming at mimicking physics. The arguments given are superficial, such as the fact that Vilfredo Pareto was an engineer before becoming an economist or that economists sometimes refer to epistemology, a discipline which took its main inspirations from physics.

But the thesis does not stand under scrutiny. Analogies between the physical world and theories on the one hand, the economic world and theories on the other hand, were hardly ever made. When they were and meant to have some weight, it was by people who were not really contributing to economics, but had fuzzy ideas about transpositions from the first field to the second, transpositions that were never really pushed through.

Mathematics is a common stock, available to scientists of all disciplines. When they draw from it, economists do not look at whether physicists are making similar drawings. They simply think it is advisable for tackling their own problems. The fact clearly appears when we study, for instance, how the theory of general economic equilibrium and its mathematization emerged from literary explorations of the same subject.

3 Interfaces between economic research and mathematics

Let us now turn attention to non-trivial mathematical problems faces by economic research. We do not need here to make a complete survey of such problems, but rather to examine a sample, of which each a piece is interesting in itself and which as whole gives a proper idea of diversity of problems. The sample will be drawn from three fields of economic research: general economic equilibrium, econometric methodology, interactive rationality (or equivalently the theory of games).

(1) Considering general economic equilibrium here is a ”must” for at least two reasons: the central role it plays in economics, the fact that it provides the case that led mathematicians to realize, in the late 1950s, that some problems in economic research were not mathematically trivial. Dealing fully with this case would take a long time. Here I must try and simply exhibit the nature of the hard problems. They belong to two parts of the theory, which will be taken in turn: the theory as providing a framework for the positive study of the effects of changes in exogenous factors, the theory as explaining the relations between economic efficiency and reliance on the price system.

General equilibrium is meant to apply to the simultaneous determination of a number of variables. Those may be symbolically represented by the couple (q, p), where q calls to mind quantities and p prices. The determination involves exogenous factors, for which we use in the same spirit the symbol z. The mathematical system derived from the theory will here be written:

$$F(q, p; z) = 0 \tag{1}$$

In principle it ought to show how q and p depend on z. But in order to find conditions under which the system does provide the determination, in order to know what are then the properties of q and p as functions of z, implicit function theorems do not suffice.

In short, the reason is a particular combination of specificity, but also generality and high dimensionality. The system, represented here as (1), is derived from a representation of market conditions and from an analysis of behaviors of consumers and producers. It is not necessarily made of ordinary equations, because for instance the analysis may lead to multivalued demand or supply functions, or to market inequalities rather than equalities, It is of course not linear, and very much so, but in a specific fashion following from hypotheses made about the microeconomics of the situation.

If the system is specific and special, it is nevertheless not fully specified, in the sense that it depends on many parameters or other features that the theory does not want to particularize. The aim is to derive general properties, applying to wide classes of cases, under hypotheses to be found.

Finally, the objects of analysis are meant to be very numerous: many goods and services, many producers and many consumers. Development of the theory even requires theorists to go beyond the case of finite numbers, either because goods of different qualities or dates of availability have really to be considered as different goods, and qualities vary continuously or dates are unbounded in the future, or because interesting results may hold only for infinitely large numbers of agents.

Although general equilibrium systems were posed since the time of Walras, their rigorous study was lagging. In 1950 it was not yet known under which conditions the standard systems could be proved to have a solution, or better a unique solution, under which conditions this solution would vary continuously as a function of exogenous factors, and so on. Answers could not be directly found in the existing mathematical literature.

The challenge was met by a rather small group of mathematical economists, who had to discover appropriate hypotheses, to work out proofs, to relax as much as possible the hypotheses, to consider increasingly general systems better suited to the development of the theory, and so on. In some rare cases the proofs involved only elementary mathematics. More often they called on recently established mathematical theorems such as Kakutani fixed point theorem (proved in 1941, used for existence of general equilibrium in 1954), or Sard theorem (proved in 1942, used for proving the generic finiteness of the number of equilibria in 1970). Some highly regarded mathematicians were attracted by this activity and spent for the purpose periods of their career in economics departments.

In the following decades a lot of attention was devoted to continuity and differentiability of the relation between z and q or p. Also important was to know what kind of restriction was imposed on this relation by the economic model.

(2) Existence of general equilibrium is an answer to a puzzle, which has occupied economists since the beginning of their science: in complex societies like ours, how are the division of labor, production, exchange and consumption arrive at without some directing agency to ensure that all individual actions are mutually consistent? The answer given by the theory is never complete since known models and properties depend on assumptions, which may appear more or less realistic but will never in fact be perfectly fulfilled.

Besides mutual consistency of individual actions, a second major puzzle is efficiency of the outcome in the use of scarce resources for the satisfaction of human needs. Efficiency of actual market economies is, of course, not perfect but has turned out to be definitely superior to that of alternative economic systems. What are the reasons? Again the answer is not complete but is the subject of an important part of general equilibrium theory.

This part accepts the concept of Pareto efficiency (within the set of physically feasible allocations of resources, an equilibrium is said to be Pareto efficient, if it is such that we could not make a consumer better off without making any other consumer worse off). The answer to the puzzle is given by ”the two theorems of welfare economics”. The first theorem asserts that a competitive equilibrium is Pareto efficient. The second theorem asserts that any Pareto efficient equilibrium can be supported by a price system. I would have to be more precise on the formalization in order to explain what is exactly meant by ”a competitive equilibrium” (which requires perfect competition and neglects the existence of ”external effects”) and by ”supported by a price systems” (which excludes collusive behavior). I shall limit attention here to the mathematics of the second theorem.

This mathematics is made of two parts. For the second one, we find the required price system by use of duality theorems for convex sets in linear spaces, as given for instance by Stefan Banach[‡]. For the first one we just have in many cases to show how individual convex sets defining the physical constraints on activities of each agent aggregate into a global convex set. We may have to reach the same global convexity by a different route, which does not require individual convexity, but the rather that agents are infinitely many and individually small (loosely speaking). This may be achieved by a limit argument on an economy with finite numbers of agents in given categories, the numbers all increasing simultaneously to infinity. This may be achieved also in a model with atomless measures of agents [§].

The duality theorem to be used in the second part of the proof could often be found in the mathematical literature; but it was not always so. In particular I had to build a rather delicate argument in order define the price system that would apply to intertemporal economies with an unbounded horizon, a feature which corresponds to common sense and to cases that are interesting for economic theories, such as that of stationary economies [¶]. The same problem attracted attention during at least the two following decades.

(3) Econometric methodology may be considered ad belonging not only to economics but also to mathematical statistics. Considered as a subclass of the latter discipline, econometrics deals with specific problems because

---

[‡] S. Banach, Theorie des operations lineaires, Warsaw, 1932; reprinted by Hafner, Nerw York

[§] R. Aumann, Markets with a continuum of traders, Econometrica, vol. 32 (1964), p. 39-50

[¶] E. Malinvaud, Capital accumulation and efficient allocation of resources, Econometrica, vol. 21 (1953), p. 233-266, and vol. 30 (1962), p. 570-573 of particular features of economic data and of economic models. Moreover econometricians with a much wider bearing than simply on econometrics.

I am choosing here, without modesty I am afraid, two examples of the latter category on which I worked in the past. This choice may simply reflect the fact that my research and teaching turned away from econometric methodology thirty years ago and that I am now ignorant of a part of the field. But it also seems to me that recent cases of mathematical problems, of which they are many, are less demonstrative because less easy to pose and to grasp, or concerning narrower classes of economic models and data.

Mathematical statistics often considers ”large samples” of data; roughly speaking those are cases in which the data base, the stochastic model and the inductive procedure are such that limit arguments, assuming an indefinitely increasing number of observation, give a good approximation for the properties of procedure. By definition the rest of mathematical statistics deals with ”small samples”. Present research in econometrics frequently concerns large samples and the asymptotic theory of estimation or tests in the many cases that quantitative economists have to tackle. When it concerns small samples, so often relevant because the data bases are not rich in numbers of observations, it most often uses Monte Carlo simulations because the mathematics of the problem is too untidy. The first example will here be drawn from large sample theory, the second one from the set of small sample problems which can be exactly solved.

Least squares regression provides a common method of estimation of unknown parameters form observation of variables linked by a relation in which these parameters occur. Let us consider the following case. A sample of T observations exists on an (endogenous) variable x_t, and a vector z_t of (exogenous) variables $(t = 1, 2 ... T)$. The stochastic model is:

$$x_t = g(z_t, a^0) + \varepsilon \tag{2}$$

where g(.,.) is a known function, a^0 is a vector of parameters, known to belong to a set A and the random variables ε,are unobservable but known to have a nil mathematical expectation and to be independently and identically distributed. The least squares estimate a_T of a^0 is computed by minimization of:

$$\sum_{t=1}^{T} [x, - g(z_t, a)]^2 \tag{3}$$

with respect to a in A.

In 1960 the properties of this estimator were well characterized for the case in which g was a linear function of a and A was a linear manifold. An asymptotic theory also existed for the general non-linear case under the assumption that the number T of observations would increase to infinity. But this theory was weak because it assumed, rather than proved, the consistency of the estimator, i.e. the property that the random estimate at would (weakly or strongly) tend to the value a^0. It was possible to exhibit cases contradicting the hypothesis. General conditions had then to be found [‖], on A, the sequence of the vectors z^t, the function g and the probability distribution of the disturbance ε_t.

(4) The mathematization of the study of inductive procedures to be used by economists sharply increased in the 1940s, particularly under the impulse of a research institution located at the University of Chicago, the ”Cowles Commission for Research in Economics”. General linear stochastic models with ”simultaneous equations” were defined, as well as methods for the estimation of their coefficients. Two alternative approaches of mathematical statistics could provide the foundations for this research. Econometricians could refer to maximum likelihood estimation and to its form in the case where the random disturbances would be normally distributed; in large samples asymptotic theory could even dispense with this last hypothesis. Econometricians could also refer to the Gauss-Markov theory of linear estimation. In both approaches generalization of known results were required in order to cover the cases considered by economists.

Reference to the Gauss-Markov theory looked to me as particularly attractive, when I started teaching econometrics in 1950s, because that was a small sample theory and some of its properties were robust with respect to the distribution of disturbances. In order to show what mathematical problem had to be solved, let me recall the bare bones of the theory in its most general and compact formulation.

In the n-dimensional Euclidian space the realization x of a random vector has been observed. The mathematical expectation of this vector belongs to a known linear subspace L. Its variance-covariance matrix Q is known. The Gauss-Markov estimator of the mathematical expectation is obtained by projection of x on L along a direction conjugate of L with respect to Q. The Gauss-Markov theorem asserts that the estimator is unbiased and efficient in the class of all linear unbiased estimators.

In 1960 the theory was not established for the most general formulation. In order to suit the needs of econometrics it had to cover the case of singular covariance matrices Q and to define in general what was meant by ”a direction conjugate of L with respect to Q"; It had also to define what was meant by efficiency of the estimator. This could be done by introduction of the concept of ”concentration ellipsoid” for random vectors[**]. The efficiency property was then that the concentration ellipsoid of the Gauss-Markov estimator was contained in that of any other linear unbiased

---

[‖] R. Jennrich, Asymptotic properties of non-linear least squares estimation, Annals of Mathematical Statistics, vol. 40 (1969), p. 633-643; E. Malinvaud, The consistency of nonlinear regressions, Annals of Mathematical Statistics, vol. 41, n. 3, June 1970, p. 959-969

[**] E. Malinvaud, Methodes Statistiques de l’Econometrie, Dunod, Paris, 1964; chapter 5; English translation published by North Holland, Amsterdam (1966)

estimator.

(5) The third field chosen here as example of interfaces between economics and mathematics, the theory of games, brings to the forefront a different kind of mathematical contribution to economic knowledge, one whose main interest may be conceptual clarification of issues involving a complex logic.

The theory of games indeed studies how each player should act to the best of her or his interest, knowing that other players are also acting to the best of their interests and are also knowing that she or he does the same. Such an interactive rationality should have a large explanatory power in economics because, almost definition, economic behavior is motivated by the pursuit of interests and because, in bargaining or otherwise, each agent often has to be conscious of the fact that other agents are also pursuing their interests.

Indeed, since the time of Cournot, economists have recurrently proposed models of such simultaneous rational behavior in various market structures belonging to what was called ”imperfect competition”. Recently attention was more and more often brought to asymmetries in the informations held by different agents when contracts between them were carried out; this feature is important in insurance, in crop-sharing or in the many other examples of so-called principal-agent relations.

Mathematicians also were attracted by the difficulty of finding out what to recommend to players in a game. The name of Emile Borel comes to mind, and still more that of John von Neumann. Actually the main founding book of the theory of games, authored by the latter, is a vivid example of interface between mathematics and economics since it was also authored by an economist and has a revealing title [††].

Outside observers know that the theory of games had an active but difficult life throughout the last fifty years. It certainly had to solve well posed mathematical problems, of which I shall give an important example in a moment. But more often it had to offer formalizations for problems that appeared at first to be indeterminate: the headache of students of interactive rationality indeed is to image the exact definition of the concept, or concepts, of solution which will be enlightening. Experience shows that the interesting concepts depend on the context in which agents are placed; these concepts are not immediately obvious and are progressively revealed as the context is studied, over and over again by various theorists. So, the theories of games, of imperfect competition, of economic behavior faced to asymmetries of imperfect competition, of economic behavior faced to asymmetries of information, result from intricate to and fro movements between discussion of cases and more or less general models; the discipline may look like zoology.

[††] J. von Neumann and O. Morgenstern, The theory of Games and Economic Behavior, Princeton University Press, (1944)

Structuring categories appeared along the process: cooperative and non-cooperative games, single and repeated games,... Pure cases put landmarks: the two-person zero-sum game, the prisoner dilemma,... General concepts of solution stood as particularly robust: Nash equilibrium, the core, the Shapley value ... It would be interesting to examine this development and to characterize in it the role of economics, as opposed to other fields of applications. This would obviously require time, much more than I could devote to the subject. I am confident, however, that eventually we should see that my discipline provided the most stimulating field for many advances.

I shall end in presenting a theoretical issue in economics that is well analyzed by the theory of games. It has long been thought that competitive imperfections tend naturally to disappear in economies where agents are numerous and individually small; perfectly competitive behavior then becomes rational for consumers and producers. The intuition is now supported by a number of formal results, which give precise meanings to the property and precise conditions for it to hold.

Formalization of an economy in which interactive rationality would imply perfect competition may use one or the other of the two devices already mentioned here when the efficiency of the general equilibrium was discussed: either the economy is the limit, when the numbers of agents increase to infinity, of a finite economy with given categories of agents, or the population of agents is a continuum with atomless measure. The first proofs of the respected property used a particular solution concept, ”the core”, in order to represent the effect of interactive rationality. But it was later found that the property was fairly robust with respect to the choice of the solution concept; for instance, it also holds with ”the Shapley value”[‡‡].

The example so appears to be not only important in itself but also more widely revealing of the role of mathematics in the elaboration of economic knowledge. It provides a good case on which to end this paper.

Ernest Malinvaud
CREST - Dept. de la Recherche
Bat Malakoff 2, Bureau 2031 - Timbre J310
15 Bd Gabriel Peri
92245 MALKOFF CEDEX

[‡‡] For an history of the problem and more generally for an account of the interface between the theory of games and economics, see R. Aumann, "Game theory", entry in the New Palgrave Dictionary of Economics. Macmillan, London (1987)