SPIRALLING SOLUTIONS IN LIMITING PROFILES OF COMPETITION-DIFFUSION SYSTEMS

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Several physical phenomena can be described by a certain number of densities (of mass, population, probability, ...) distributed in a domain and subject to laws of diffusion, reaction, and *competitive interaction*. Whenever the competitive interaction is the prevailing phenomenon, the several densities can not coexist and tend to segregate, hence determining a partition of the domain *(Gause's experimental principle of competitive exclusion (1932))*. As a model problem, we consider the system of stationary equations

$$\begin{cases} -\Delta u_i = f_i(u_i) - \beta u_i \sum_{j \neq i} g_{ij}(u_j) \\ u_i > 0 \ . \end{cases}$$

The cases $g_{ij}(s) = \beta_{ij}s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij}s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

We will undertake the analysis of qualitative properties of solutions to systems of semilinear elliptic equations, whenever the parameter β, accounting for the competitive interactions, diverges to infinity. At the limit, when the minimal interspecific competition rate $\beta = \min_{ij} \beta_{ij}$ diverges to infinity, we find a vector $U = (u_1, \cdots, u_h)$ of functions with mutually disjoint supports: *the segregated states*: $u_i \cdot u_j \equiv 0$, for $i \neq j$, satisfying

$$-\Delta u_i = f_i(x, u_i) \qquad \text{whenever } u_i \neq 0 \ , \quad i = 1, \ldots, h,$$

We will review the known results and focus on long range interactions and spiralling solutions in the non symmetrical case: $(\beta_{ij} \neq \beta_{ji})$.