

What's in an envelope? Operator theory and numerical ranges

The numerical range of an $n \times n$ matrix A is defined by

$$W(A) = \{\langle Ax, x \rangle : x \in \mathbb{C}^n, \|x\| = 1\}.$$

We give a brief overview of the numerical range of an operator and discuss several open questions that motivate this study. We will discuss the geometry of the numerical range, focusing on a particular class of operators known as compressions of the shift operator. We provide a discussion of algorithms for computing the numerical ranges of these operators as well as several results describing when the numerical range is elliptical.