Fixed Points, Semigroups and Rigidity of Holomorphic Mappings

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Abstract

There is a long history associated with the problem of iterating nonexpansive and holomorphic mappings and finding their fixed points, with the modern results of K. Goebel, W.-A. Kirk, T. Kuczumow, S. Reich, W. Rudin and J.-P. Vigué being among the most important.

Historically, complex dynamics and geometrical function theory have been intensively developed from the beginning of the twentieth century. They provide the foundations for broad areas of mathematics. In the last fifty years the theory of holomorphic mappings on complex spaces has been studied by many mathematicians with many applications to nonlinear analysis, functional analysis, differential equations, classical and quantum mechanics. The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system: $\frac{dx}{dt} + f(x) = 0$, where x is a variable describing the state of the system under study, and f is a vector-function of x. The study of such systems when f is a monotone or an accretive (generally nonlinear) operator on the underlying space has recently been the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems.

In this talk we give a brief description of the classical statements which combine the celebrated Julia Theorem of 1920, Carathéodory's contribution in 1929 and Wolff's boundary version of the Schwarz Lemma of 1926 with their modern interpretations for discrete and continuous semigroups of hyperbolically nonexpansive mappings in Hilbert spaces. We also present flow-invariance conditions for holomorphic and hyperbolically monotone mappings.

Finally, we study the asymptotic behavior of one-parameter continuous semigroups (flows) of holomorphic mappings. We present angular characteristics of the flows trajectories at their Denjoy-Wolff points, as well as at their regular repelling points (whenever they exist). This enables us by using linearization models in the spirit of functional Schroeder's and Abel's equations and eigenvalue problems for composition operators to establish new rigidity properties of holomorphic generators which cover the famous Burns-Krantz Theorem.