ON POSITIVE SOLUTIONS OF HALF-LINEAR EQUATIONS WITH THE (p, A)-LAPLACIAN AND A POTENTIAL IN MORREY SPACE

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A partial differential equation $Q(u) = 0$ is said to be *half-linear* if for any $\alpha \in \mathbb{R}$ we have $Q'(\alpha v) = 0$, whenever $Q'(v) = 0$. So, half-linear equations satisfy the homogeneity property of linear equations but not the additivity.

In this talk we present qualitative positivity properties of quasilinear equations of the form

$$Q'_{A,p,V}[v] := -\mathrm{div}(|\nabla v|_A^{p-2} A(x)\nabla v) + V(x)|v|^{p-2}v = 0 \qquad x \in \Omega,$$

where Ω is a domain in $\mathbb{R}^n$, $1 < p < \infty$, $A = (a_{ij}) \in L^\infty_{\mathrm{loc}}(\Omega; \mathbb{R}^{n\times n})$ is a symmetric and locally uniformly positive definite matrix, V is a real potential in a certain local Morrey space (depending on p), and

$$|\xi|_A^2 := A(x)\xi \cdot \xi = \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \qquad x \in \Omega,\ \xi = (\xi_1, \ldots, \xi_n) \in \mathbb{R}^n.$$

Our assumptions on the coefficients of the operator for $p \geq 2$ are the minimal (in the Morrey scale) that ensure the validity of the local Harnack inequality and hence the Hölder continuity of the solutions. For some of the results of the paper we need slightly stronger assumptions when $p < 2$.

This is a joint work with Georgios Psaradakis.

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