SEMILINEAR ELLIPTIC PDES ON COMPLETE MANIFOLDS WITH BOUNDARY

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Abstract

The goal of this talk is to give an overview of the results obtained over the last few years about positive solutions of the following nonlinear problem

$$\begin{cases} \Delta u + a(x)u - b(x)u^{\sigma} + c(x)u^{\tau} = 0 & \text{on } M, \\ \partial_{\nu} u + \displaystyle\sum_{i=1}^{n_H} g_i(x) u^{q_i} = 0 & \text{on } \partial M; \end{cases}$$

where $(M, \partial M, \langle\ ,\ \rangle)$ is a complete, noncompact, Riemannian manifold with boundary ∂M (not necessarily compact). Here $a(x)$, $b(x)$, $c(x)$, and $g_i(x)$ are continous functions, while τ, σ, and q_i are real coefficients such that $\tau < 1 < \sigma$. Special cases of the problem above are the Yamabe-type equations and the Lichnerowicz equations with nonlinear Neumann conditions.

We have obtained *a priori* estimates, comparison results, Liouville-type theorems, and existence of positive solutions. The presence of a nonempty boundary ∂M forces to avoid curvature assumptions (this is due to the lack of effective curvature comparison results in this case), indeed, the results have been obtained under suitable spectral hypoteses and assumptions on the volume growth of geodesic balls at infinity. One of the main tool that we have developed has been a refined version of the weak maximum principle adapted to the case of a manifold with boundary.

These results have interesting applications to Geometry (the prescribed curvature equation, the Yamabe problem, constancy of harmonic maps) and General Relativity (construction of initial data sets for Einstein-scalar field theories).

Most of the results have been obtained in collaboration with Marco Rigoli (Università degli Studi di Milano).

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